Overview of action balance and source terms

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Atmospheric and Oceanic Science

Covered in this lecture:

- Third generation wave models and the Action Balance Equation.
- General solution techniques.
- Physics:
  - Deep water.
  - Shallow water.
- Data assimilation.
- Outlook / future.

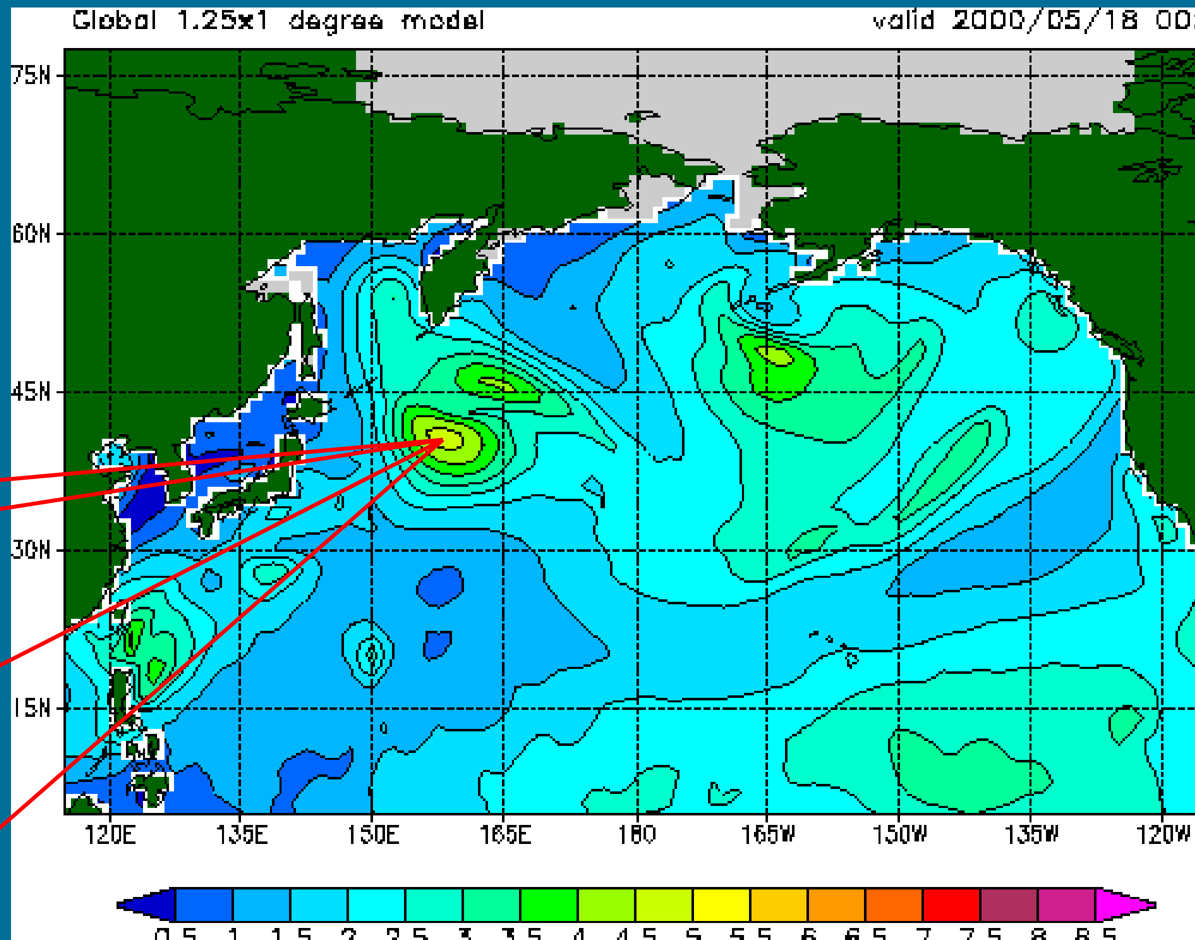
The basis of all spectral random-phase models is the spectra action balance equation

$$\frac{\partial N(\mathbf{i})}{\partial t} + \nabla_{\mathbf{x}} \bullet (\mathbf{c}_g + \mathbf{U}) N(\mathbf{i}) + \nabla_{\mathbf{i}} \bullet \mathbf{c}_{\mathbf{i}} N(\mathbf{i}) = \sum S(\mathbf{i})$$

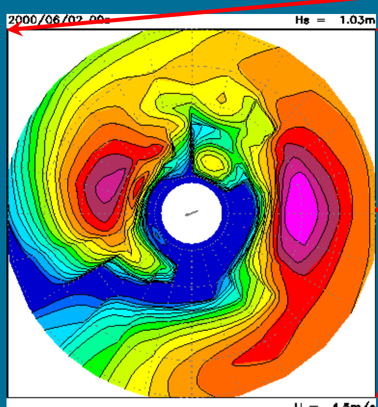
$N(\mathbf{i}) = \dfrac{F(\mathbf{i})}{\sigma}$	Action density spectrum in terms of the energy spectrum and intrinsic frequency.
$\mathbf{i}$	Spectral phase space (wavenumber, frequency, direction, 2D).
$\nabla_{\mathbf{x}}, \nabla_{\mathbf{i}}$	Divergence operators.
$\mathbf{c}_g, \mathbf{c}_{\mathbf{i}}, \mathbf{U}$	Characteristic and current velocities.
$\sum S(\mathbf{i})$	Sources and sinks of energy / action.

3G models

significant wave height (m)



Spectral description of wave field at each grid point



Ocean wind wave models are classified by their treatment of the source terms.

$$\frac{\partial N(\mathbf{i})}{\partial t} + \nabla_{\mathbf{x}} \bullet (\mathbf{c}_g + \mathbf{U}) N(\mathbf{i}) + \nabla_{\mathbf{i}} \bullet \mathbf{c}_{\mathbf{i}} N(\mathbf{i}) = \boxed{\sum S(\mathbf{i})}$$

- First and second generation models use observed spectral shapes and sustained spectral energy levels to infer effects of physical processes.
- Third generation (3G) model parameterize all physical processes explicitly (degrees of freedom = # of spectral bins), not imposing spectral shapes or energy levels.
  - Exception is unresolved high-frequency part of wave spectrum (shape defined, level is not).

3G models have become the standard for ocean wave modeling.

- Much more expensive that 1G and 2G models.
- Much more versatile than 1G and 2G models.
  - Less need for site-specific tuning.
  - New science directly applicable.
- Still too expensive for some applications ?
  - Coupled modeling.
  - Commercial applications.

There are many 3G models available.

- The most widely used models are WAM, SWAN and WAVEWATCH III.
  - WAM is the original.
    - No strong central support system since the mid 1990's.
    - Now semi-proprietary versions.
  - SWAN and WAVEWATCH III actively supported and freely available.
- Several commercial models:
  - OceanWeather, DHI, …..
- Several research models
  - Exact-NL, …..

Solution techniques

$$\frac{\partial N(\mathbf{i})}{\partial t} + \nabla_{\mathbf{x}} \bullet (\mathbf{c}_g + \mathbf{U}) N(\mathbf{i}) + \nabla_{\mathbf{i}} \bullet \mathbf{c}_{\mathbf{i}} N(\mathbf{i}) = \sum S(\mathbf{i})$$

- The traditional way to solve this equation is to consider it as a hyperbolic equation and march the solution forward in time. This technique is used by most models.
- The exception is the SWAN model, which traditionally considers the quasi-stationary version of the Eqs.

$$\nabla_{\mathbf{x}} \bullet (\mathbf{c}_g + \mathbf{U}) N(\mathbf{i}) + \nabla_{\mathbf{i}} \bullet \mathbf{c}_{\mathbf{i}} N(\mathbf{i}) = \sum S(\mathbf{i})$$

- This becomes an elliptical Eq., that is solved by an iterative sweeping procedure.
- Later, SWAN reintroduces instationarity …

- Features of hyperbolic approach:
  - Simple/cheap solvers for each time step (typical explicit FD formulations).
  - Explicit FD formulations result in small time steps for high-resolution models, making models expensive for typical coastal applications.

focus here

- Features of elliptical approach:
  - In stationary mode, solutions only when needed; cheap modeling for high resolution coastal applications.
  - Iterative solver for implicit problem more complicated.
  - Large time steps possible for coastal models.
  - Stability versus accuracy with large time steps.

Economical solution:

- Five dimensional problem. To save memory and to keep numerical schemes simple, use a fractional step method.

$$\frac{\partial N(\mathbf{i})}{\partial t} + \nabla_{\mathbf{x}} \bullet (\mathbf{c}_g + \mathbf{U}) N(\mathbf{i}) + \nabla_{\mathbf{i}} \bullet \mathbf{c}_{\mathbf{i}} N(\mathbf{i}) = \sum S(\mathbf{i})$$

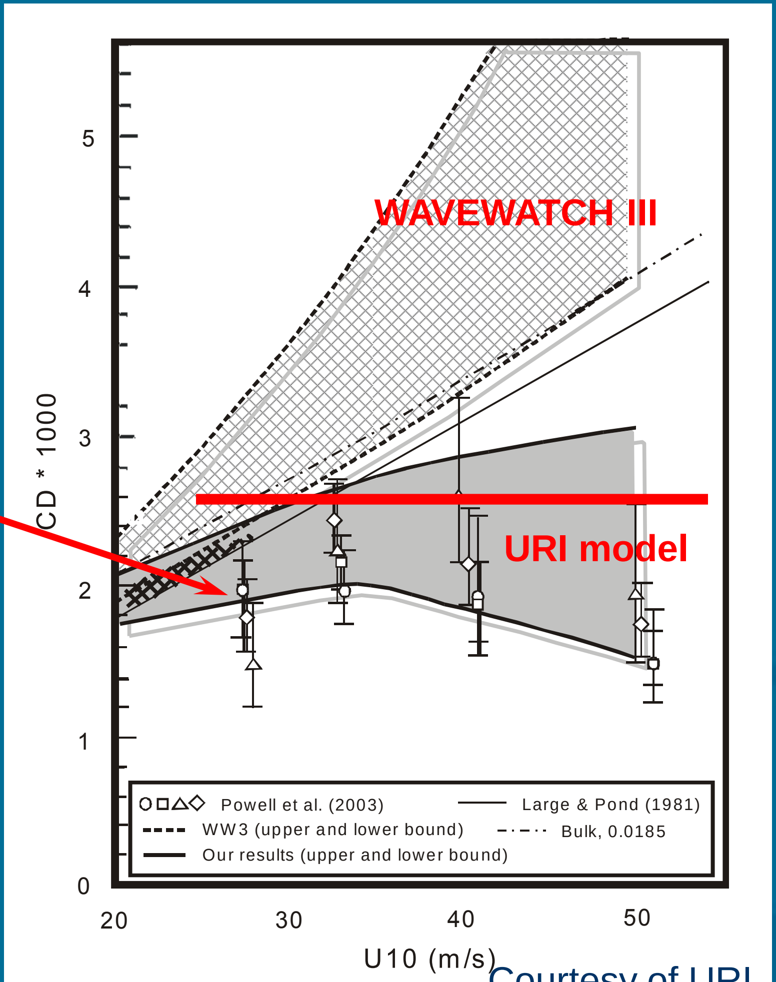
- Subsequent equations solved in WAVEWATCH III
  - Spatial propagation.
  - Intra-spectral propagation.
  - Source terms
  - Water level changes (remapping only).

Sources and sinks

$$\sum S(\mathbf{i}) = S_{in}(\mathbf{i}) + S_{nl}(\mathbf{i}) + S_{ds}(\mathbf{i}) + \ldots\ldots$$

- Wind input (linear and exponential).
- Nonlinear interactions (4-wave, resonant).
  - Not important for propagation, critical for wave growth.
- Dissipation (whitecapping).
- Many additional processes in shallow water.
  - Bottom friction.
  - Shallow water wave breaking.
  - Triad interactions.
  - …

- Reasonably well established, but hard to measure accurately. Do you believe in negative input ?
- Critical issue: Stresses in high wind regime:
  - Observations ….. Powel et al (2003)
  - Models:
    - URI, RSMAS, SMU, …..
- Engineering solution:
  - Cap on C_d.
- Long term include sea spray to constrain all fluxes,



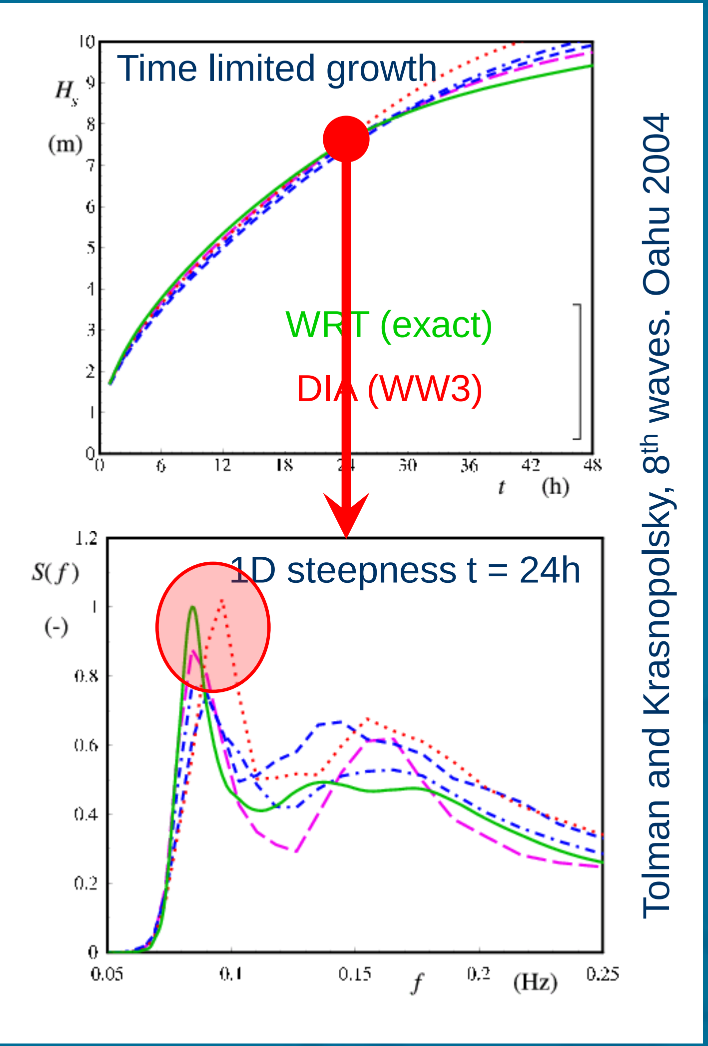
Courtesy of URI

Nonlinear interactions are at the center of 3G models.

- Without parameterizations, third generation models are not possible.
- Exact interactions are well known, but very expensive to compute.
  - Exact-NL, WRT, RIAM, ….
  - Shallow water also possible, but more complicated.
- Discrete Interaction Approximation (DIA) makes 3G models feasible for operations.
- DIA has serious flaws (following slides), replacements have been long coming, but are reaching maturity
  - MDIA, SRIAM, TSA, NNIA, …..

Physics (quadruplet interactions)

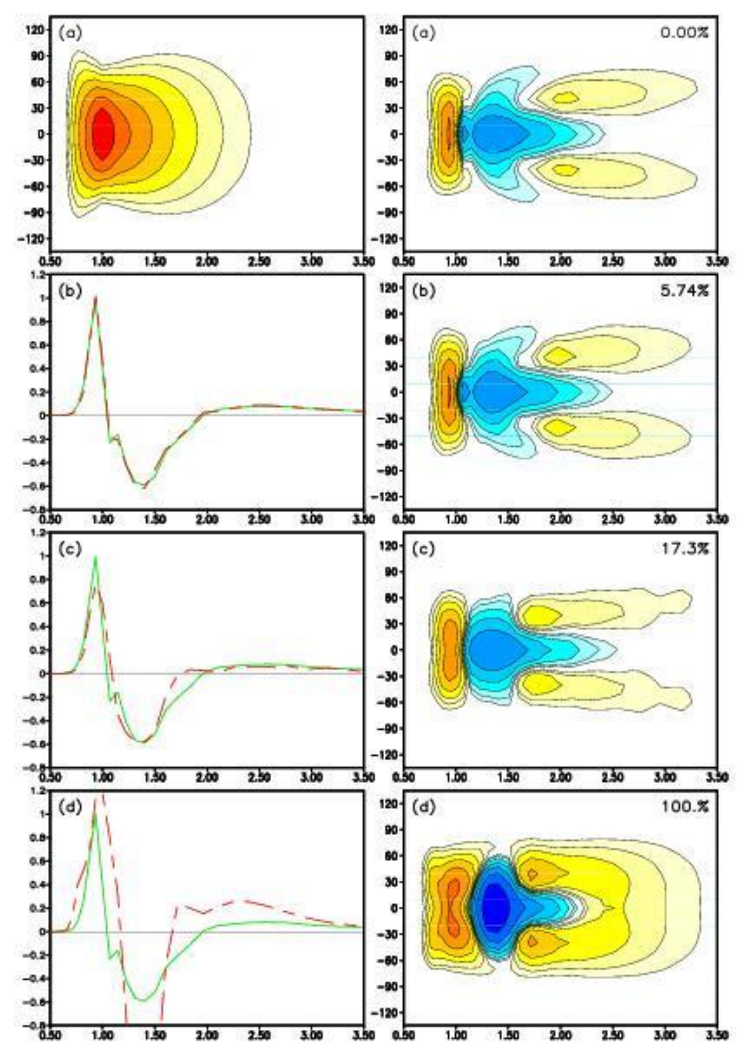
- Success of interactions from DIA with respect to wave heights is misleading,
- Near identical wave heights from various interaction approaches can lead to large differences in spectral shape.
- WAVEWATCH III present approach has issues with location of spectral peak.



Tolman and Krasnopolsky, 8th waves. Oahu 2004

Physics (quadruplet interactions)

- Computation of interactions for test spectra shows that much better results can be obtained than with DIA.
- Better interactions do not necessarily result in better model integration.
  - VDIA is unstable !
- Holistic optimization based on full model integration is needed.
- Shallow water scaling is needed.



Exact (WRT)

MDIA N = 4

VDIA N = 1

DIA

Tolman, *Ocean Mod.*, 2004(**6**)

Physics (whitecapping dissipation)

In most early 3G models, this is the tuning term and not much more …….

- Original in WAM "weak in the mean" (= linear) only.
- WAVEWATCH III default (Tolman and Chalikov 1996) :
  - Peak frequency based on wind sea only.
  - Linear at low freq., local nonlinear at high freq.
  - Based on wind stress (shallow, currents ?).
- Since then, much progress from Australia
  - Saturation based on wave breaking in groups,
  - Explicit estimation of breaking occurrence and intensity.
- NCEP transitioning to Ardhuin et al (2010) and additions.

- Can only be expected to come to full fruition with a better parameterization of S_{nl}.

Recently, an additional deep-water source term has become more prevalent.

$$\sum S(\mathbf{i}) = S_{in}(\mathbf{i}) + S_{nl}(\mathbf{i}) + S_{ds}(\mathbf{i}) + S_{sd}(\mathbf{i}) + \ldots\ldots$$

- Swell dissipation is essential to be included in operational models.
- Dissipation with time scale of days to weeks is small compared to whitecapping, but notable over life of swell.
  - In WAVEWATCH III as negative wind input needed to eliminate model biases in tropical Pacific (T&C 1996).
  - Inconclusive from 1960's "waves across the Pacific" experiment.
  - Recent SAR obs. (models) from Ardhuin *et al*.

- So far we have looked at deep water physics only.
- Shallow water physics are additional, and can only work properly if the deep water part is taken care of,

but,

- Most people that interact with waves live on the coast, and therefore are affected by the shallow water effects …

Note that :

- Refraction and shoaling are part of propagation and are relatively well understood.

Physics (bottom interaction)

- Wave-bottom interactions occur in intermediate water depths (outer shelf up to coast), and have been long been investigated (Shemdin et al, 1978, review paper) ….
  - Bottom friction most generally addressed:
    - Simple linear JONSWAP expression,
    - or drag-law style (Madsen et al.).
    - Interaction through oscillatory turbulent boundary layer:
      - No impact of mean currents other than kinematics.
      - Rough turbulent boundary layer means that friction is determined by physical bottom roughness (k_N or z_0).
      - wave-induced ripples change roughness by orders of magnitude, but require sub-grid approach.

Friction cont’ed :

- Don’ts for bottom friction:
  - Interaction goes through turbulence, therefore do not add wave and mean current velocities.
  - A friction factor associated with bottom friction varies with flow conditions. Pre-describing a friction factor rather than a roughness over-estimated nonlinearity.
- Physically most sound approaches :
  - JONSWAP linear term is based on observations, but constant does not appear universal.
  - Nonlinear dissipation based on physical bottom roughness
    - Many different bottom types, including moving sediments.

Other bottom interaction mechanisms:

- Percolation:
  - Similar to JONSWAP linear friction term.
- Wave-mud interactions:
  - Many different models.
  - May influence dispersion relation of spectral components.
  - Possible interplay with three-wave interactions ?
  - Hot item in US with New Orleans problems.

Shallow water (depth-induced) breaking is dominant dissipation mechanism in surf zone.

- Orders of magnitude more intense than whitecapping.
- All parameterizations have root in Battjes and Janssen (1978) parameterization.
- Key issues:
  - Behavior on different slopes (including flat bottom).
  - Integration with whitecapping.
  - Phase resolving versus phase averaging.

- Note: This source term is the "safety valve" for limiting wave heights in shallow water.
- Note: This source term is key in coupling wave and surge / inundation models.

Triad resonant interactions can only occur in extremely shallow water.

- Shifting energy to higher frequencies.
- Avoid unrealistic long periods on the beach.
- Simple parameterizations are available in, e.g., SWAN.
- Key issues:
  - Work reasonably well on beaches, but not on extended shallow areas.
  - Is “no parameterization” better than a poor parameterization?
  - Phase resolving versus phase averaging; is this a local or cumulative process?

Other physics parameterizations have been suggested

- Effects of rain on wave.
  - Effecting capillary waves and therefore remote sensing.
- Scattering of waves due to interaction with smaller scale bottom features.
- Is there a need for a new source term for wave energy dissipation in case of wave blocking?
- Advanced wave-ice interactions.
  - Models are available with a complexity comparable to S_{nl}.

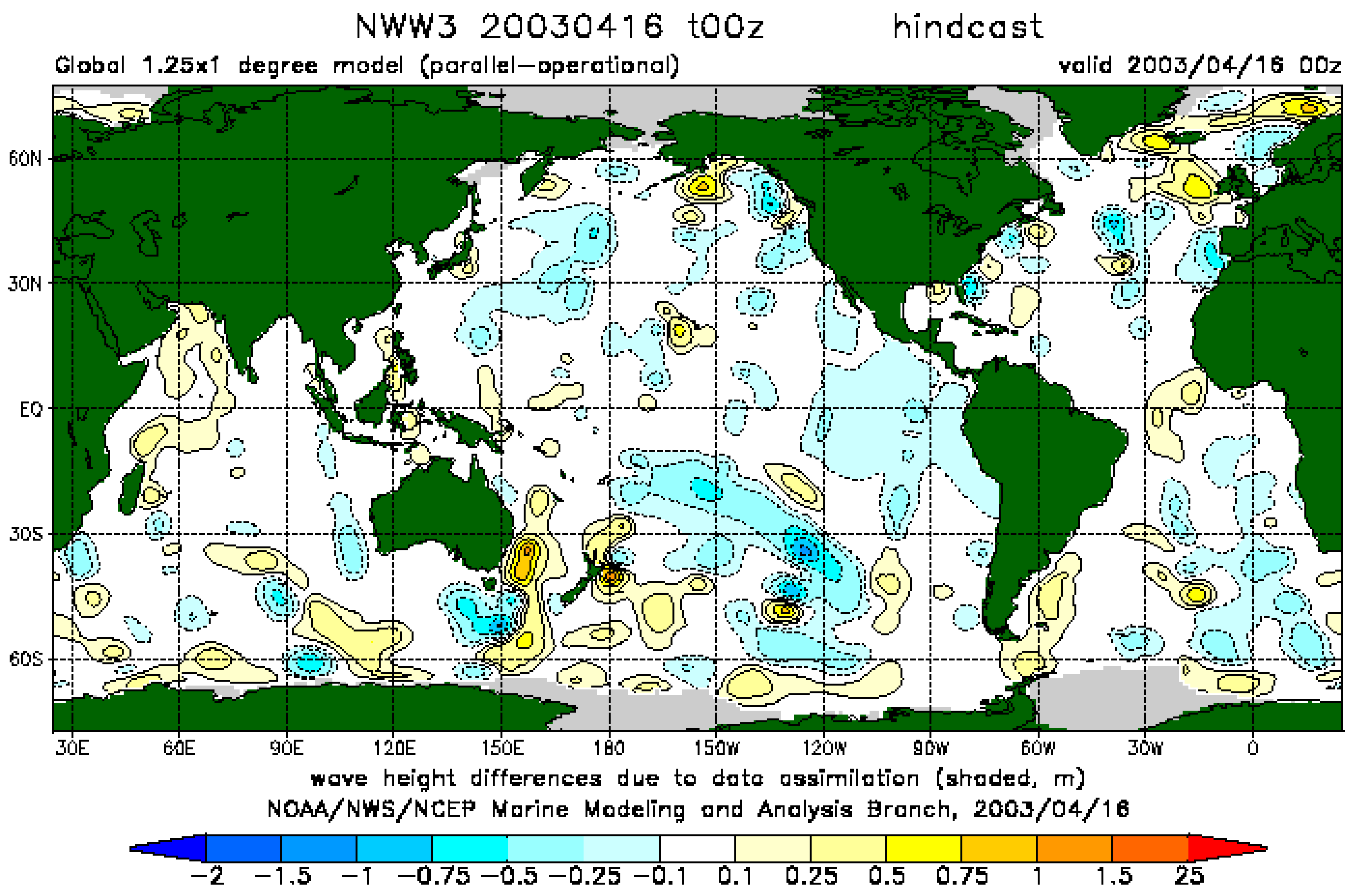
- We expect to see effect of oil and other slicks on waves to be modeled explicitly in the future.

This is not a weather model !!!!

- Wind waves represent a forced and damped problem.
  - This is not an initial value problem like a weather or ocean model.
- Therefore, a very good wave model can be build without any data assimilation.
  - Weather and ocean models are critically dependent upon initial conditions and therefore on data assimilation.
- Due to the forced and damped nature, retention time of data into a wave model is limited by definition:
  - Typically 12-36 hours in terms of impact on wave height in typical operational models.
  - May be much smaller if a coastal model uses coastal buoy data.
  - Up to weeks for Pacific swell.

There is not enough data.

- There is no data-dominated analysis.
- Quality of analysis is by definition misleading.



Is there a need for DA in wave models?

- Yes, our operational customer needs the best possible model, and DA will improve short-term forecasts.

- There are political reasons to do DA.
  - Establish real-time operational use of data sources.
  - This justifies existence of data sources.
  - Data sources are critical for off-line development and quality assessment of operational models.
    - Off-line use does not carry weight with administrators like real-time use does.

Historical approaches to DA:

- Green's function approach by Hasselmann et al:
  - Use inverse model to estimate errors in wave generation.
  - Correct model forcing to minimize wave generation errors.
  - Re-run model with improved forcing.
- Features:
  - Expensive, therefore going back only hours, not days to improve forcing.
  - Requires enough data to constrain error in full forcing field.
  - Assumes perfect wave model.
- Not used in any operational model.

History cont’ed:

- Updating of forcing by describing wind fields as splines (De Valk et al.)
- Similar in concept to previous page, but cheaper.
- Not used in any operational model.
- Both approaches have advantages that wind sea is modified consistently with the wind field, and, if corrections of forcing are used in forecast too, will remain in the model system. Swell is perturbed automatically.

History cont’ed:

- Analysis of wave height based on altimeter and buoy data, using OI or 2DVar methods.
- Features:
  - Analyze wave height, but need to update spectrum. This is by definition a subjective process.
  - Using techniques from other fields (SST and others), without acknowledging wave physics.
  - Original work at KNMI (De Las Heras et al): DA cannot correct for errors in numerical propagation scheme.
    - Contrary to assumption of perfect model in previous methods.
  - Economical.

OI / 2DVar cont’ed:

- Can be augmented with assimilation of swell observation (spectral buoy data, SAR data).
- More advanced methods using wave physics are slowly coming to fruition (Greenslade et al.)
- Can potentially be used in combination with ensemble information.
  - Ensemble Kalman Filtering.
  - Hybrid 2DVar-Ensemble methods,
- Used operationally world wide.

Physics:

- Do we need nonlinear propagation in the models?
  - Willebrand (1974) nonlinear shallow water propagation.
  - Nonlinear propagation at wave blocking (Chawla and Kirby).
  - Extension to 3G models.

- The more elementary question is: does the sine-base random phase model suffice?
  - Stokes based descriptions (Chalikov, Janssen).
  - Is phase information needed, particularly in depth-limited water?
  - Evolution to 4G models.

Physics cont'ed:

- The 3(4) deep water source terms seem to work reasonably well in most models, however, a structural re-evaluation is due.
  - NOPP project starting 2010 to upgrade these source terms to state-of-the-art.
    - See previous source term slides.
    - Note default in WAVEWATCH III 3.14 is from 1995!
    - Operational updates at NCEP based on model version 4.xx.
    - Adding the swell dissipation as an additional process.
      - Wind input and/or dissipation ?

Physics cont' ed:

- The description of triad interactions is highly unsatisfactorily in 3G models.
  - Integrate with quadruplets (NOPP funded).
  - Phase averaged versus phase resolving.
  - Can this be done purely local ? and
  - Can this be separated from breaking ?
- Breaking is described by two or even three processes: (whitecapping, surf breaking, blocking dissipation).
  - Separate description prone to "double counting".
  - Integration in single process desirable.
    - Assuming that all breaking represent similar physical processes.

The end

End of lecture