



State of the art of wave modeling

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- Wave modeling history.
- Third generation models.
- Governing equations.
- Solution techniques
- Numerics.
- Physics.
- Data assimilation.
- Ensembles.
- The future.



Many good books and publications are available. Here are some subjective selections:

- Historical
 - JONSWAP, SWAMP, and SWIM.
 - WAM book (Komen et al, 1994)
- Undergraduate level:
 - Leo Holthuijsen.
 - Ian Young.
- Non-English research access
 - Igor Lavrenov
- Recent journal publications
 - WISE whitepaper (*Progress in Oceanography*, 2007).



To talk about the future,
you need to know the past.

- Generally, predictions of wave conditions for D-Day (June 1944) are considered the first attempt at operational wave forecasting.

Mulberry harbors of Gold Beach June 1944.





- First operational forecasting for D-day, 1944.
 - Up to the late 1950's, parametric relation between the significant wave height and the local wind speed.
-
- Gelci et al., late 1950's, introduced spectral concept to numerical wave modeling.
 - Up to the late 1980's many models developed, using simple nonlinear interaction approximations and/or assumptions on spectral shape (first and second generation models).
 - After SWAMP study in mid 1980's, community effort to develop WAM with explicit treatment of nonlinear interactions (third generation models), essentially replacing all previous models.



- NCEP's first computer-aided wave forecast was made in 1956, producing only a wave heights and period based on present and recent local winds.
- In 1968, the system was expanded to estimate a single wind seas and a single swell (H_s , T_p).
- The first operational spectral wave was introduced at NCEP in 1985, and was based on Cardone's second generation SAIL model (first global, later regional).
- Some of these models were replaced by cycle 4 of the WAM model in 1994 and 1997.
- Development of an in-house third-generation model (WAVEWATCH III[®]) started in 1993.
- From 1998-2000 all operational wave models at NCEP have become WAVEWATCH III based.



- Forecasting requires large scale models, even for small scale coastal forecast problems:

- Additionally spectral random phase models
- Elements of large scale random phase models can be distinguished in categories are

Ray models

Grid models

- Furthermore, these models can be distinguished

Five approaches

Finite difference approaches

Finite element approaches.

- Elements of phase-resolving models start making their way into these models.

AND ANYTHING

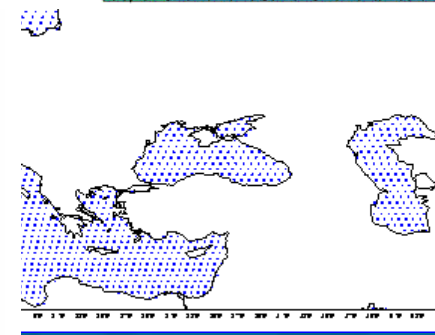
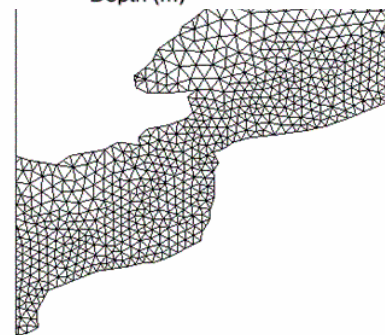
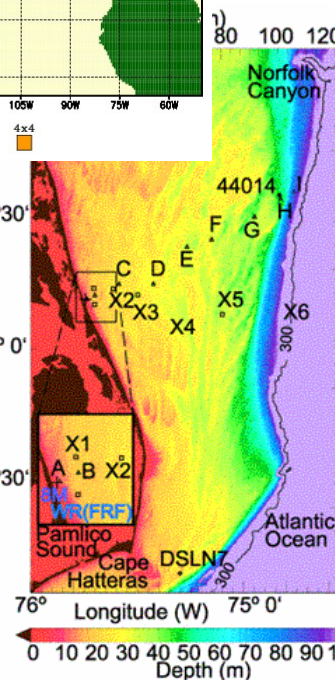
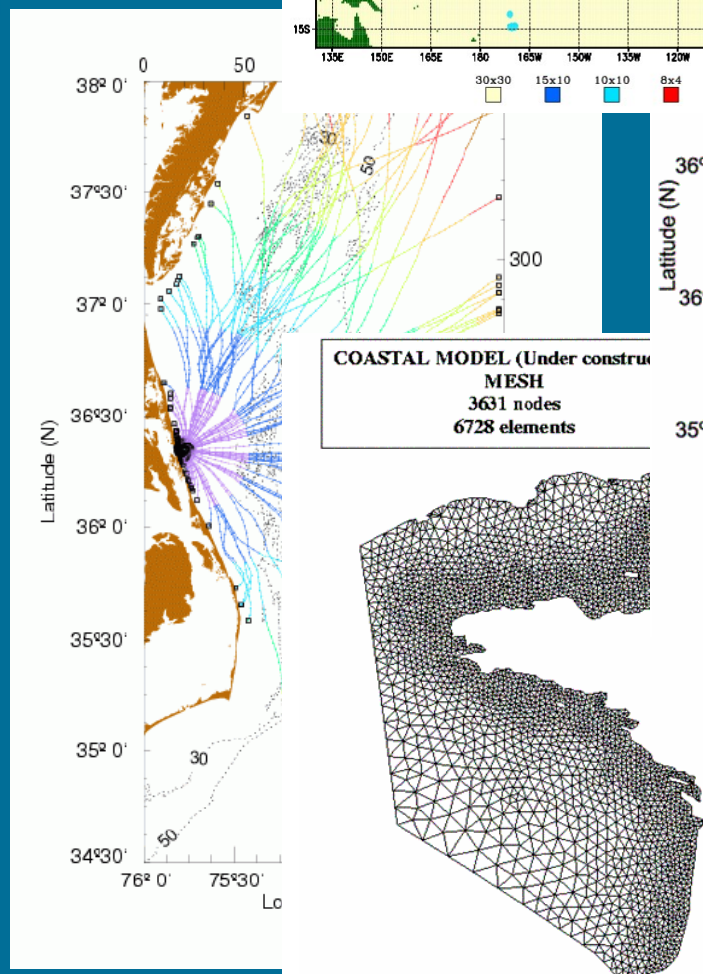
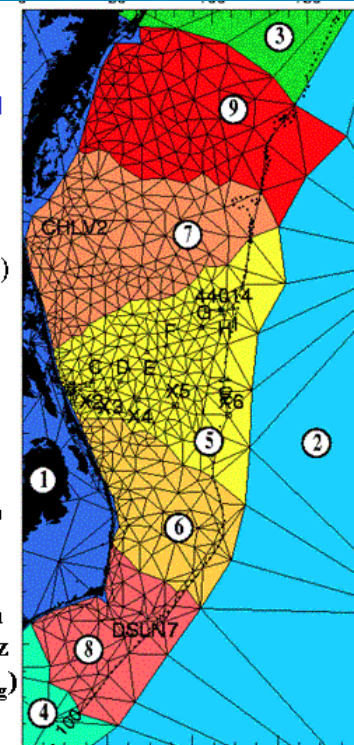
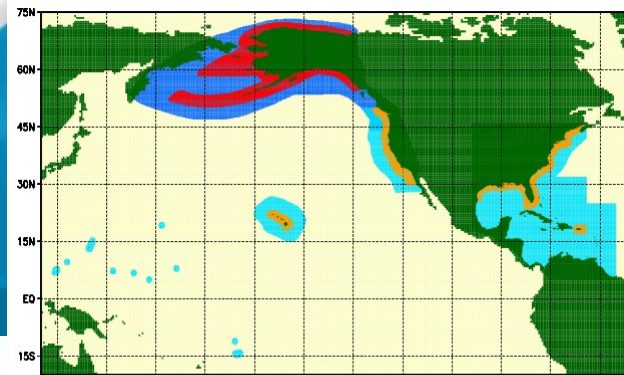
"A MATTER OF TASTE"

IN BETWEEN



D1. validation Bathymetry and CREST (JPO, May 2001) model grid

570 grid points
19 frequencies
(0.05 – 0.15 Hz)
72 directions
(5 deg. resolution)
 $\Delta t = 10$ minutes
→ 8× real time on
SGI IP27 300 MHz
(with S_{fric} and S_{Bragg})
using 850 Mb





The basis of all spectral random-phase models is the spectral action balance equation

$$\frac{\partial N(\mathbf{i})}{\partial t} + \nabla_{\mathbf{x}} \bullet (\mathbf{c}_g + \mathbf{U})N(\mathbf{i}) + \nabla_{\mathbf{i}} \bullet \mathbf{c}_i N(\mathbf{i}) = \sum S(\mathbf{i})$$

$N(\mathbf{i}) = \frac{F(\mathbf{i})}{\sigma}$ Action density spectrum in terms of the energy spectrum and intrinsic frequency.

$\mathbf{i}$ Spectral phase space (wavenumber, frequency, direction, 2D).

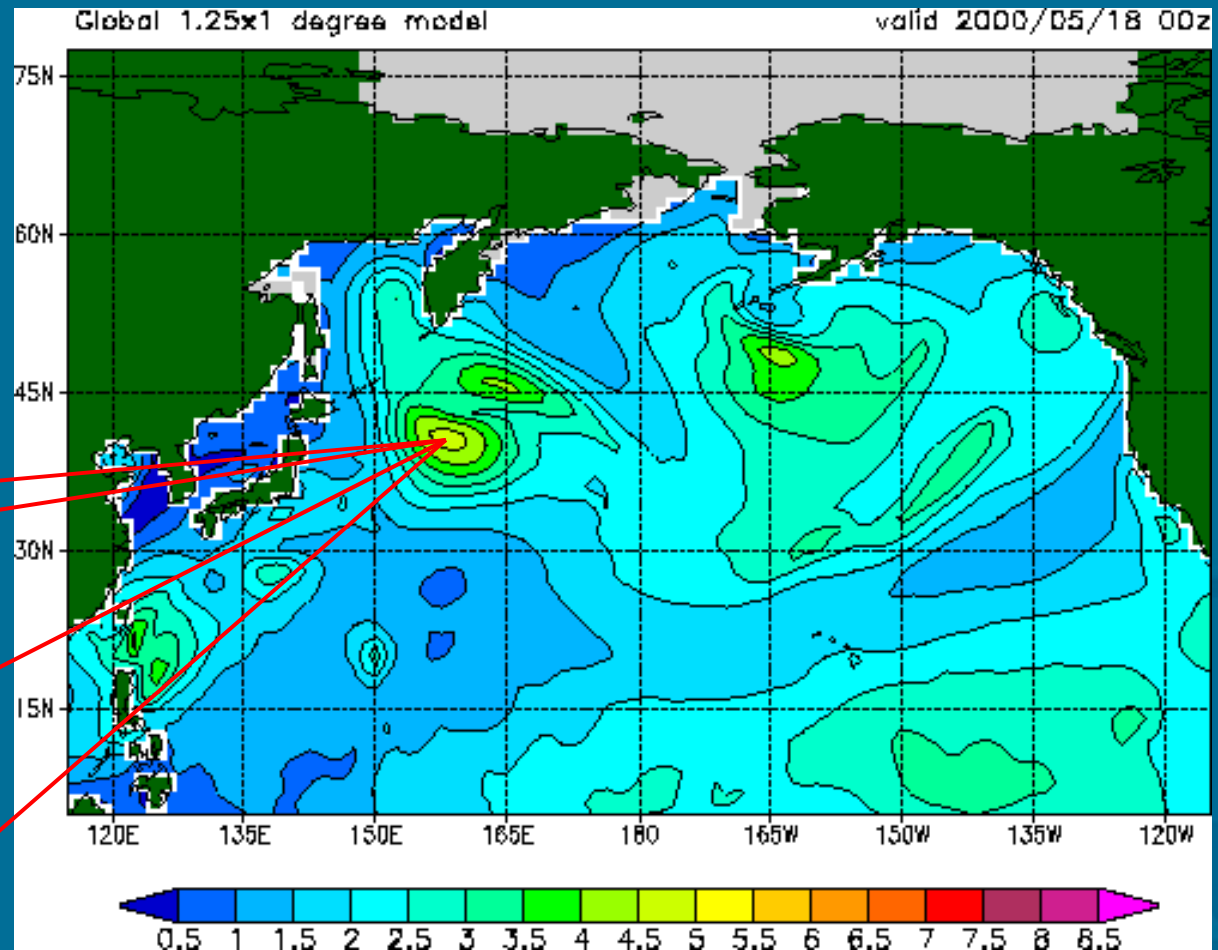
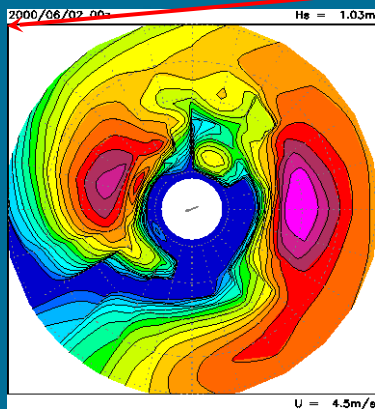
$\nabla_{\mathbf{x}}, \nabla_{\mathbf{i}}$ Divergence operators.

$\mathbf{c}_g, \mathbf{c}_i, \mathbf{U}$ Characteristic and current velocities.

$\sum S(\mathbf{i})$ Sources and sinks of energy / action.

significant wave height (m)

Spectral description
of wave field at
each grid point





Ocean wind wave models are classified by their treatment of the source terms.

$$\frac{\partial N(\mathbf{i})}{\partial t} + \nabla_{\mathbf{x}} \bullet (\mathbf{c}_g + \mathbf{U})N(\mathbf{i}) + \nabla_{\mathbf{i}} \bullet \mathbf{c}_i N(\mathbf{i}) = \sum S(\mathbf{i})$$

- First and second generation models use observed spectral shapes and sustained spectral energy levels to infer effects of physical processes.
- Third generation (3G) model parameterize all physical processes explicitly, without imposing spectral shapes or energy levels.

Exception is unresolved high-frequency part of wave spectrum (shape defined, level is not).



3G models have become the standard for ocean wave modeling.

- Much more expensive than 1G and 2G models.
- Much more versatile than 1G and 2G models.
 - Less need for site-specific tuning.
 - New science directly applicable.
- Still too expensive for some applications ?
 - Coupled modeling.
 - Commercial applications.



There are many 3G models available.

- The most widely used models are WAM, SWAN and WAVEWATCH III.

WAM is the original.

No strong central support system since the mid 1990's.

Now semi-proprietary versions.

SWAN and WAVEWATCH III actively supported and freely available.

- Several commercial models:
OceanWeather, DHI,
- Several research models
Exact-NL,



$$\frac{\partial N(\mathbf{i})}{\partial t} + \nabla_{\mathbf{x}} \bullet (\mathbf{c}_g + \mathbf{U})N(\mathbf{i}) + \nabla_{\mathbf{i}} \bullet \mathbf{c}_i N(\mathbf{i}) = \sum S(\mathbf{i})$$

- The traditional way to solve this equation is to consider it as a hyperbolic equation and march the solution forward in time. This technique is used by most models.
- The exception is the SWAN model, which traditionally considered the quasi-stationary version of the Eqs.

$$\nabla_{\mathbf{x}} \bullet (\mathbf{c}_g + \mathbf{U})N(\mathbf{i}) + \nabla_{\mathbf{i}} \bullet \mathbf{c}_i N(\mathbf{i}) = \sum S(\mathbf{i})$$

- This becomes an elliptical equation, that is solved using an iterative sweeping procedure.
- Later, SWAN reintroduces instationarity ...



- Features of hyperbolic approach:

Simple/cheap solvers for each time step (typical explicit FD formulations).

Explicit FD formulations result in small time steps for high-resolution models, making models expensive for typical coastal applications.

focus here

- Features of elliptical approach:

In stationary mode, solutions only when needed; cheap modeling for high resolution coastal applications.

Iterative solver for implicit problem more complicated.

Large time steps possible for coastal models.

Stability versus accuracy with large time steps.



Economical solution:

- Five dimensional problem. To save memory and to keep numerical schemes simple, use a fractional step method.

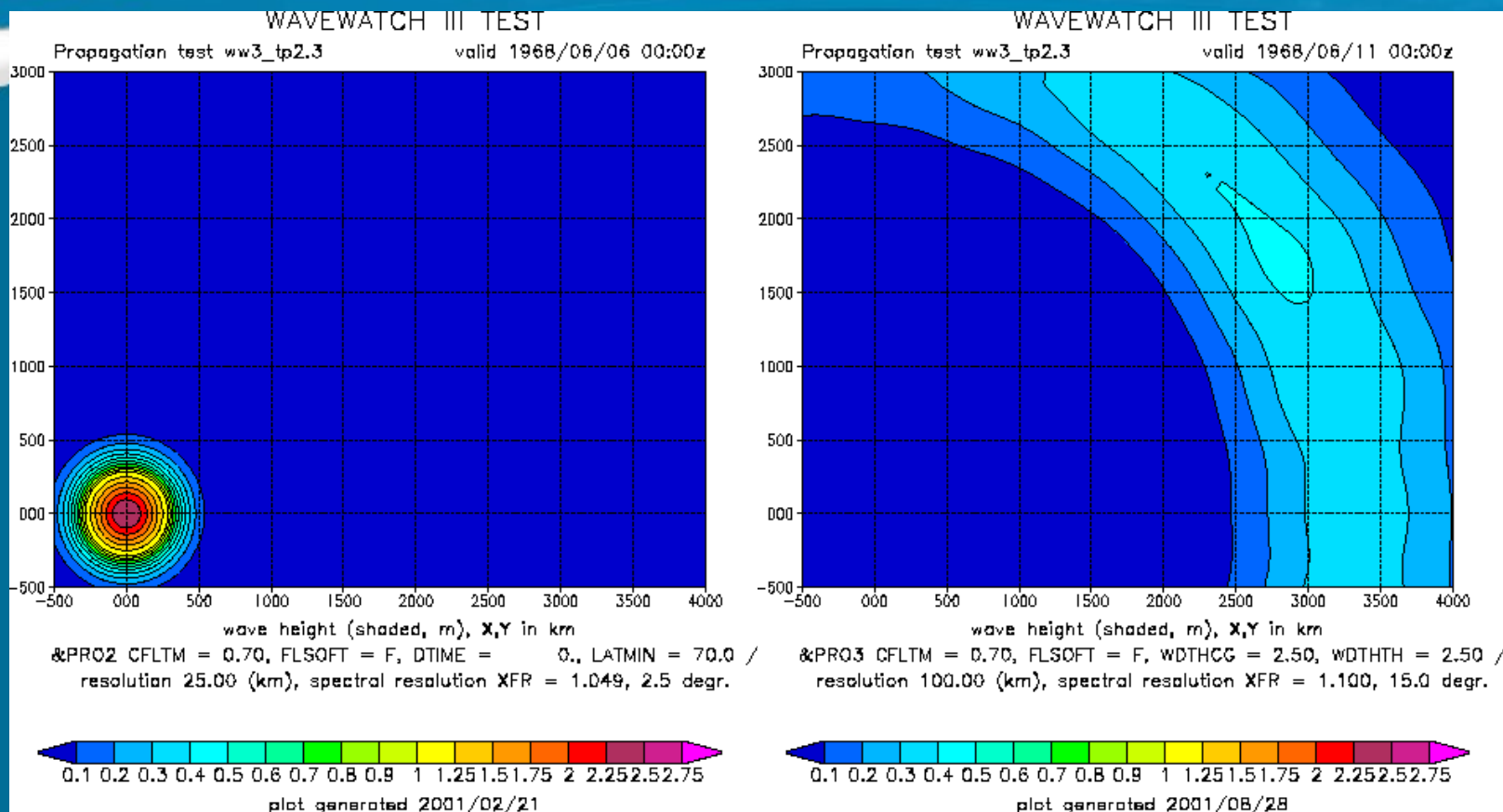
$$\frac{\partial N(\mathbf{i})}{\partial t} + \nabla_{\mathbf{x}} \cdot (\mathbf{c}_g + \mathbf{U})N(\mathbf{i}) + \nabla_{\mathbf{i}} \cdot \mathbf{c}_i N(\mathbf{i}) = \sum S(\mathbf{i})$$

- Subsequent equations solved in WAVEWATCH III
 - Spatial propagation.
 - Intra-spectral propagation.
 - Source terms
 - Water level changes (remapping only).



Many numerical issues occur:

- Most traditionally associated with (spatial) propagation.
 - Physically simple, numerically hard.
 - Positive definite advection over long distances.
 - Garden Sprinkler Effect (GSE).
 - Grid approaches.
 - Implicit versus explicit numerical methods.
 - Semi-Lagrangian in between.
 - Refraction, wave blocking.
 - Accuracy not assessed systematically.
- Source term integration often ignored, but not trivial.



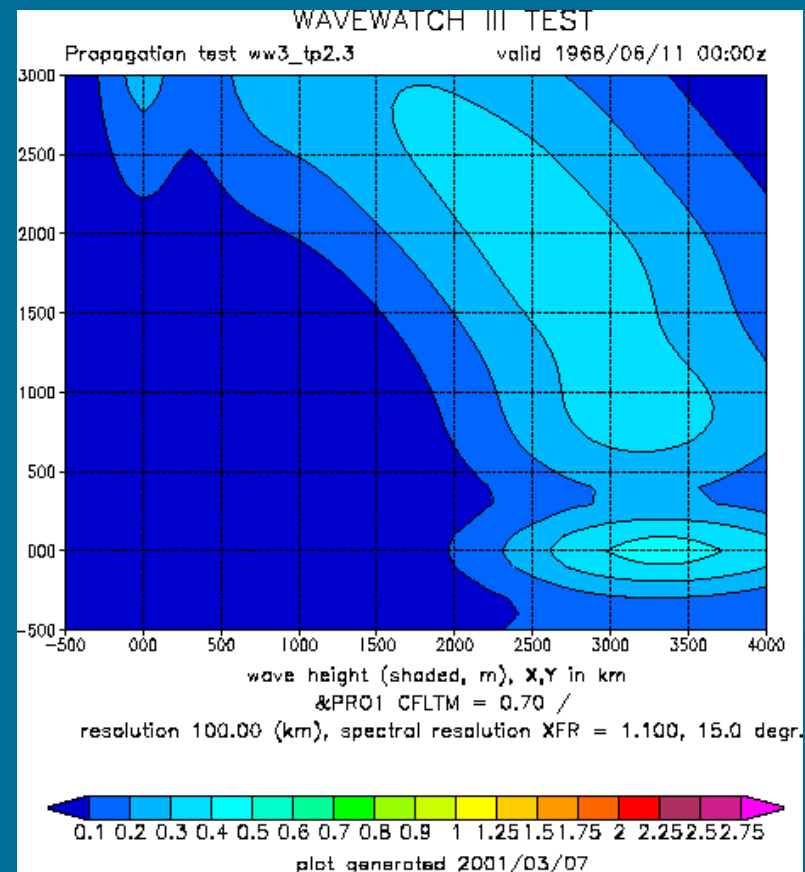
Exact solution: continuous dispersion of swell energy over a large area.

Accurate scheme gives inaccurate results (Garden Sprinkler Effect, GSE)

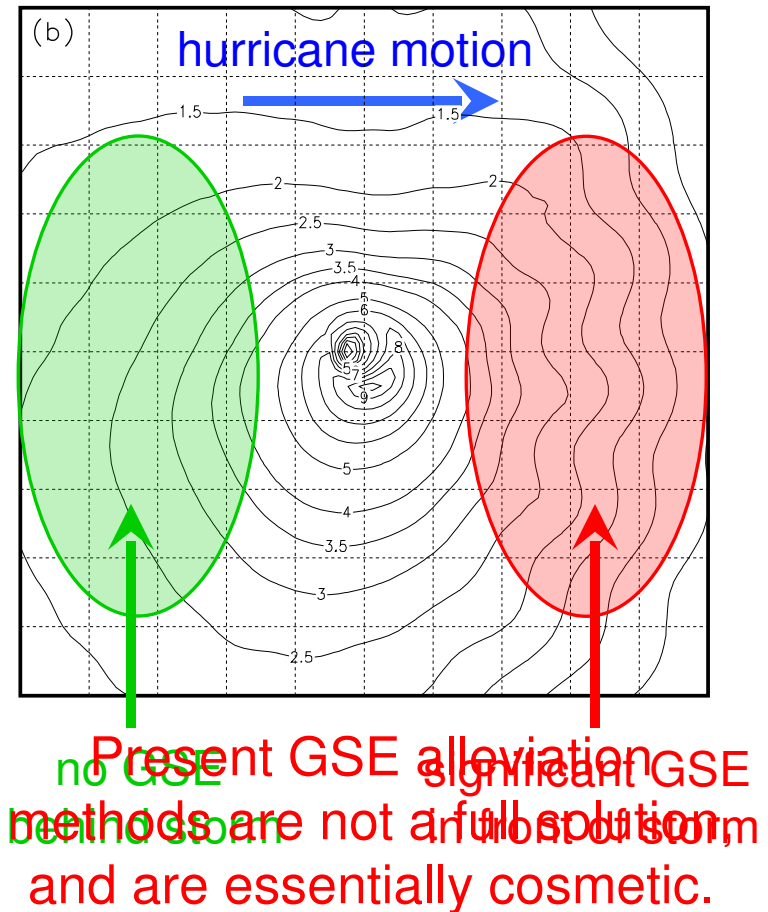
Accurate scheme + GSE alleviation!



- Several 3G models still use first order propagation schemes, suggesting that its numerical diffusion will remove the effects of the GSE.
- Unfortunately, numerical diffusion is directionally anisotropic, and therefore, the argument does not hold.



- Third order propagation schemes seem adequate for now.
- GSE alleviation methods work for now, but
 - Require tuning / not universal.
 - Is expected to show up again with higher resolution intense forcing (hurricanes).
- Explore Booij and Holthuijsen (1987) or divergent advection further.



Tolman and Alves, *Ocean Mod.*, 2005(9)



As already illustrated before, there are many ways to do spatial propagation on a grid.

- Traditionally, explicit finite differences on Cartesian or spherical grids.

Relatively simple implementation.

Implicit schemes generally not used due to dimensionality of problem.

Third order schemes adequate for swell as shown in GSE examples.

Pole in spherical grid causes problems with time steps.

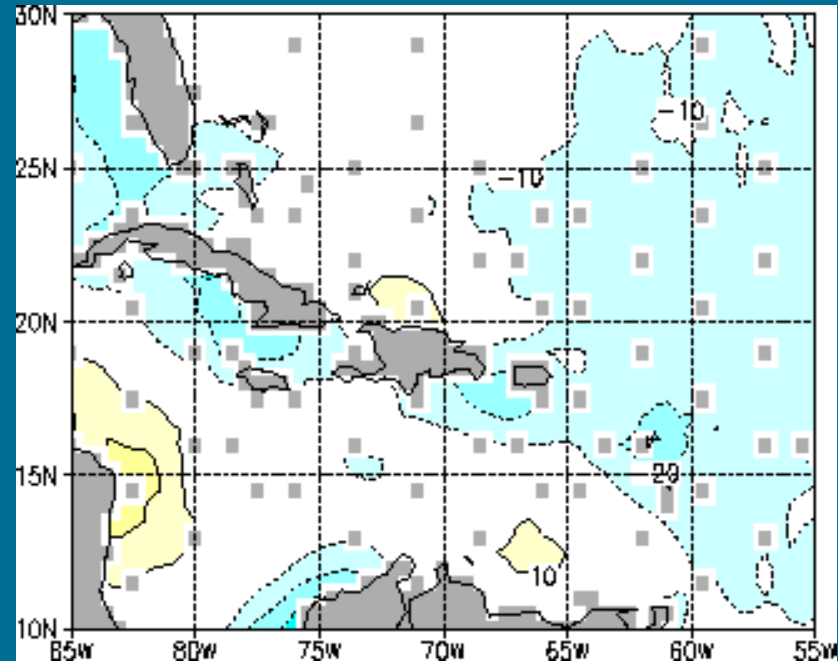
Coastal scales result in small time steps.

Difficult to resolve barrier islands and atolls adequately

Sub-grid approach now commonplace.

- In particular in regular grids, unresolved islands allow swell energy to travel without proper dissipation.
- Modeled as a sub-grid obstacle.
Now in many 3G models.
As in modeling Great Barrier Reef (Hardy et al. 2000, *Ocean Eng.*).
- Can be used for simple ice modeling too (Tolman 2002, *Ocean Modelling*)

bias with sub-grid islands

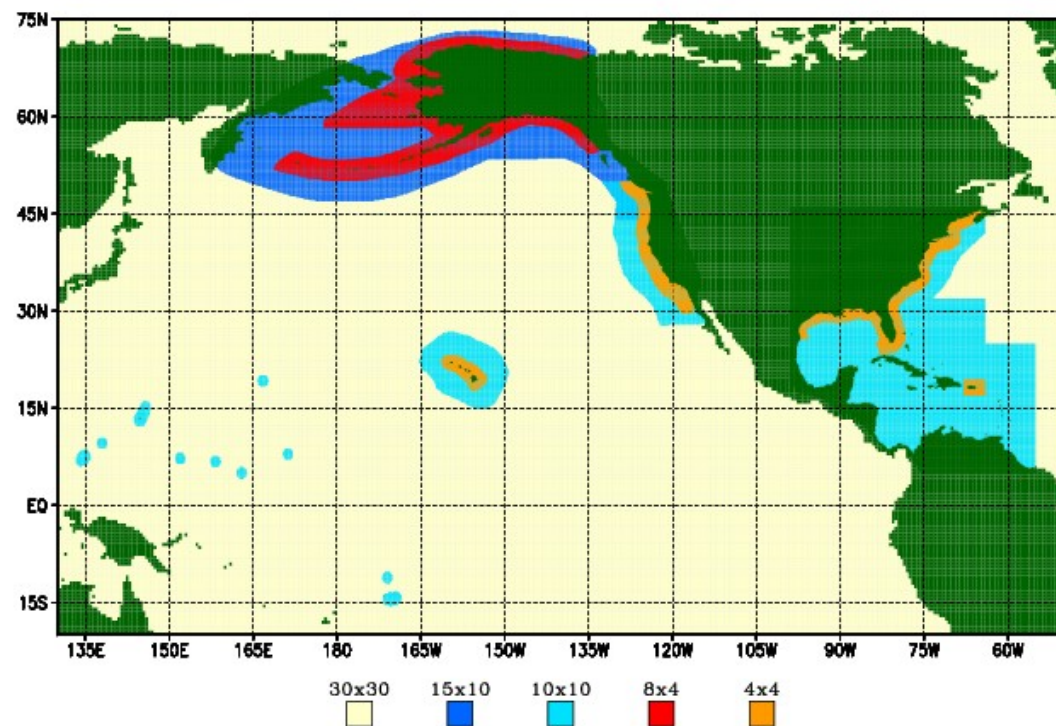
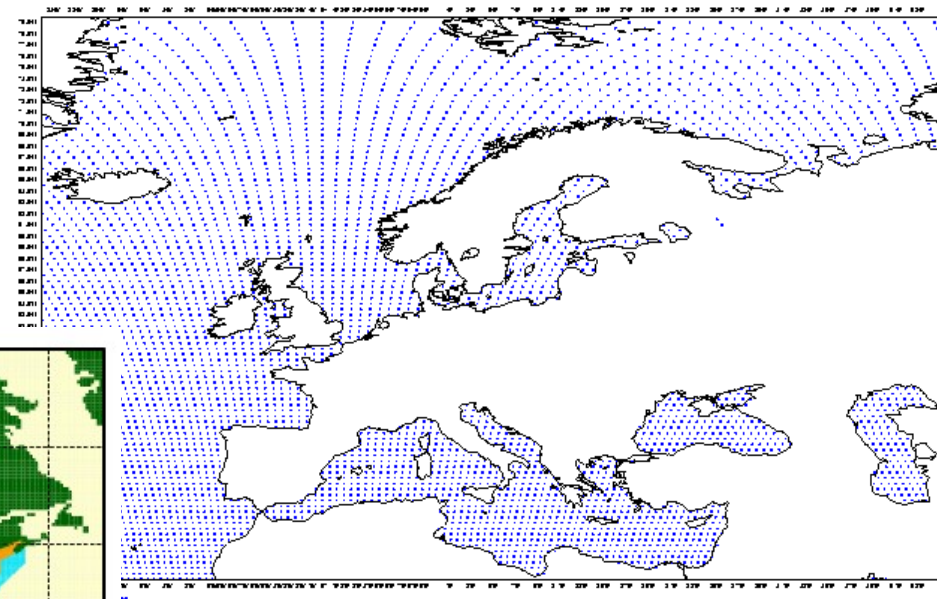


Example from NCEP Western North Atlantic model.
Bahamas and Lesser Antilles not resolved.

Refinements:

- ECWAM reduced grid (0th order scheme).
- WAVEWATCH III mosaic approach.

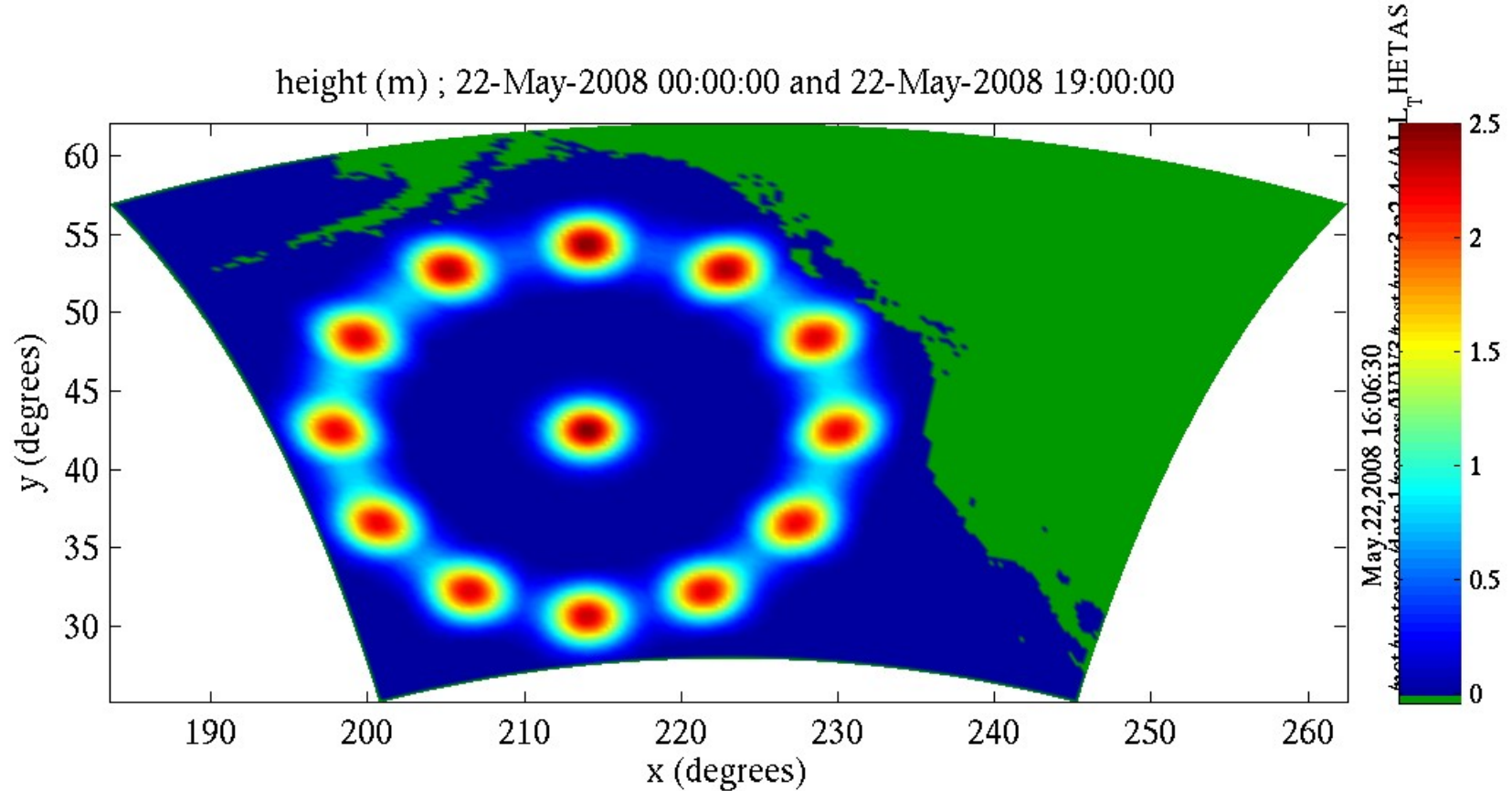
courtesy of Jean Bidlot (ECMWF)



Alternatives:

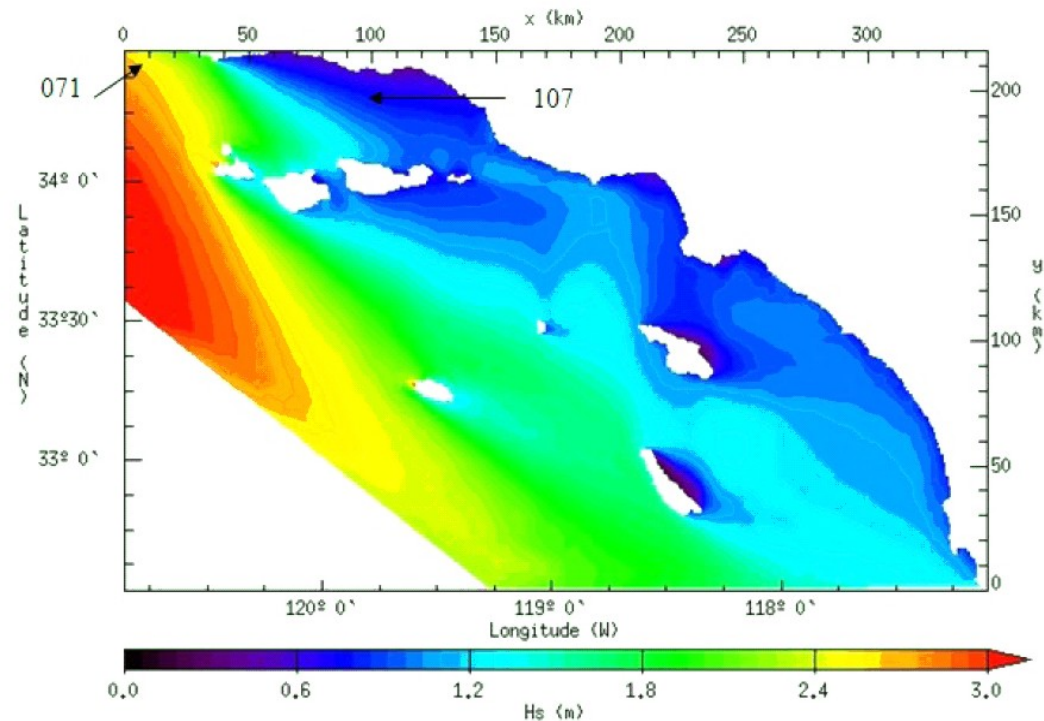
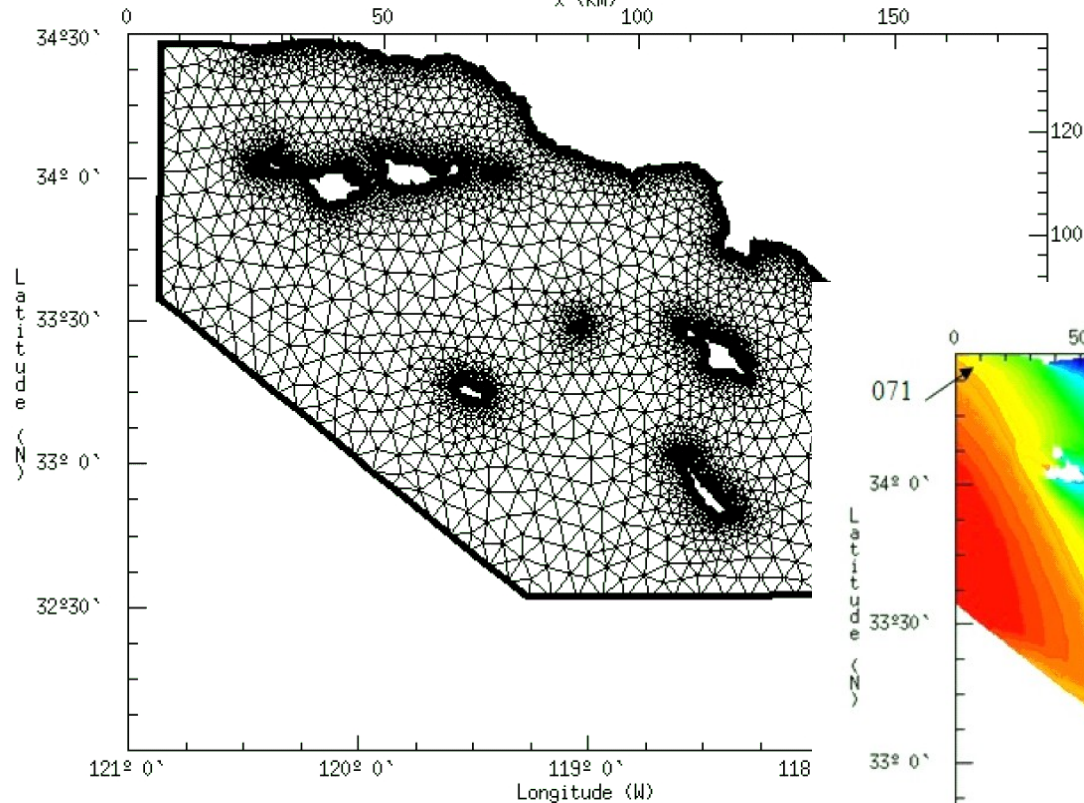
- Curvilinear grids
- Unstructured grids

Numerics (grids)



Swell propagation test on a curvilinear grid developed by Erick Rogers (NRL), from experimental WAVEWATCH III.

Numerics (grids)



Unstructured grid developed by Romain Cancouet (ENSIETA, France), based on original work by Aaron Roland included in experimental WAVEWATTCH III.



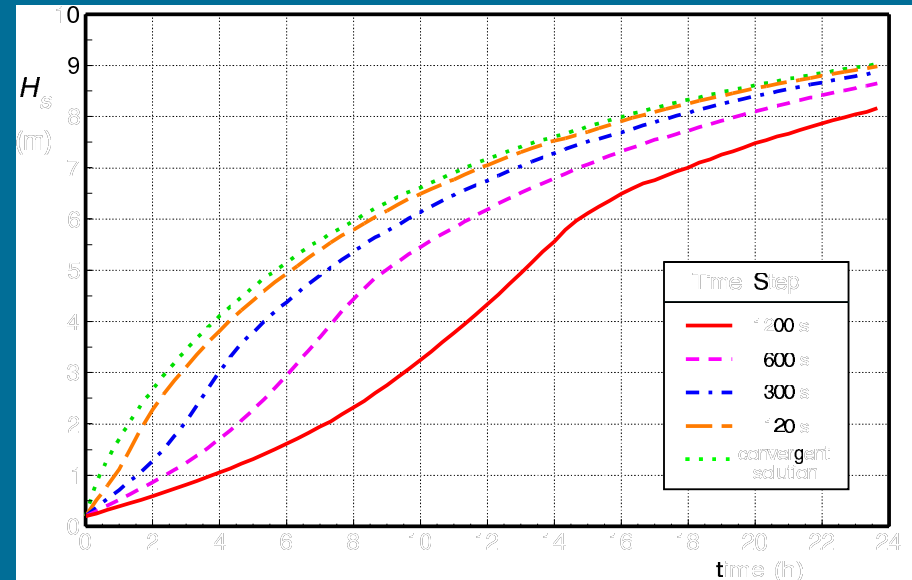
- WAM uses a limiter to allow source term integration to use large time steps.

This introduces time step dependence of growth curves.

Engineering solution provided by Hersbach and Janssen (1999).

- SWAN also uses some type of limiter.
- WAVEWATCH (II) uses dynamic time stepping instead (Tolman 1992).

Incompatible with WAM or SWAN



Example based on WAM-3 physics.

Dynamic time step closely reproduces convergent solution



Sources and sinks

$$\sum S(\mathbf{i}) = S_{in}(\mathbf{i}) + S_{nl}(\mathbf{i}) + S_{ds}(\mathbf{i}) + \dots$$

- Wind input (linear and exponential).
- Nonlinear interactions (4-wave, resonant)
Not important for propagation, critical for wave growth,
- Dissipation (whitecapping).
- Many additional processes in shallow water.
Bottom friction.
Shallow water wave breaking.
Triad interactions.
...



- Reasonably well established, but hard to measure accurately. Do you believe in negative input ?
- Critical issue: Stresses in high wind regime:

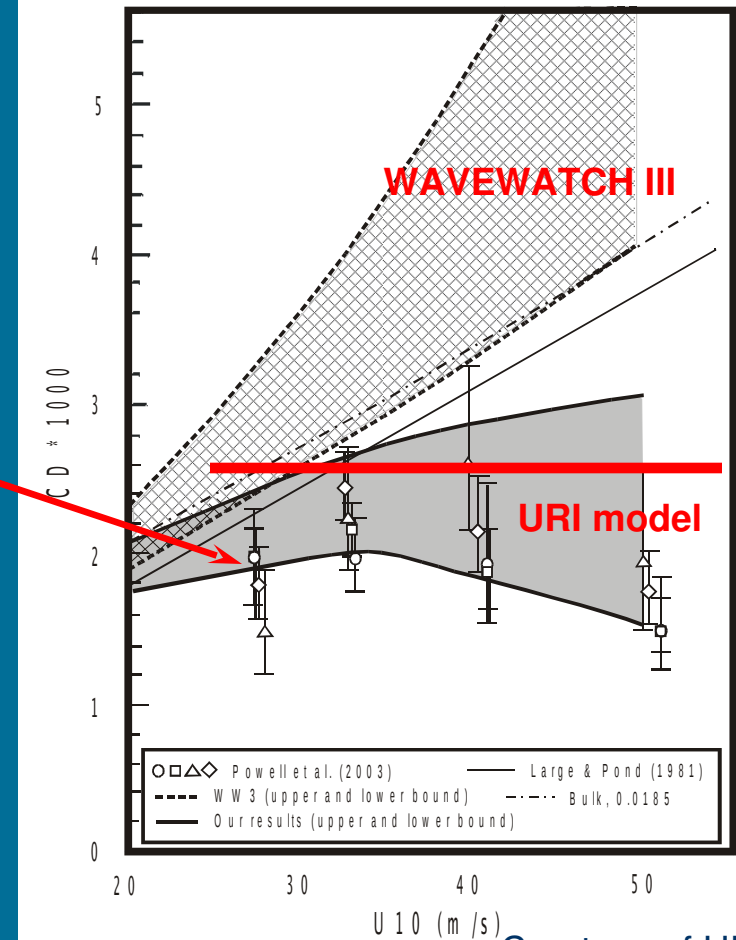
Observation

Models:

URI, RSMAS, SMU,
.....

- Engineering solution:
Cap on C_d .
- Long term include sea spray to constrain all fluxes in coupling.

Powell et al
(2003)



Courtesy of URI



Nonlinear interactions are at the center of 3G models.

- Without parameterizations, third generation models are not possible.
- Exact interactions are well known, but very expensive to compute.

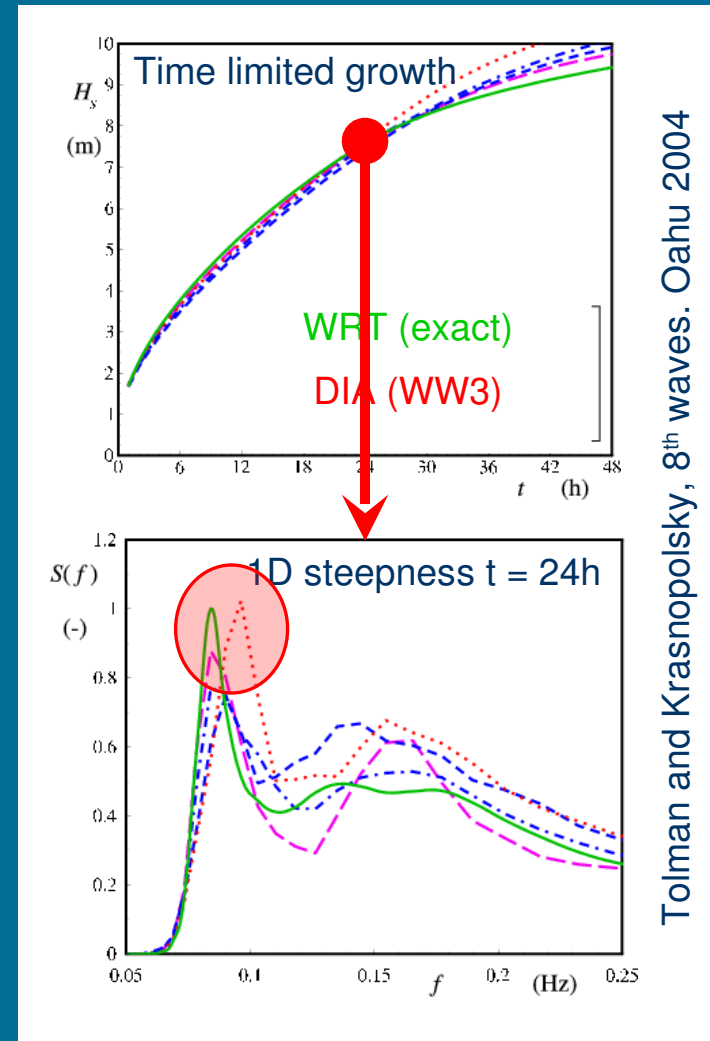
Exact-NL, WRT, RIAM,

Shallow water also possible, but more complicated.

- Discrete Interaction Approximation (DIA) make 3G models feasible for operations.
- DIA has serious flaws (following slides), replacements have been long coming, but are reaching maturity

MDIA, SRIAM, TSA, NNIA,

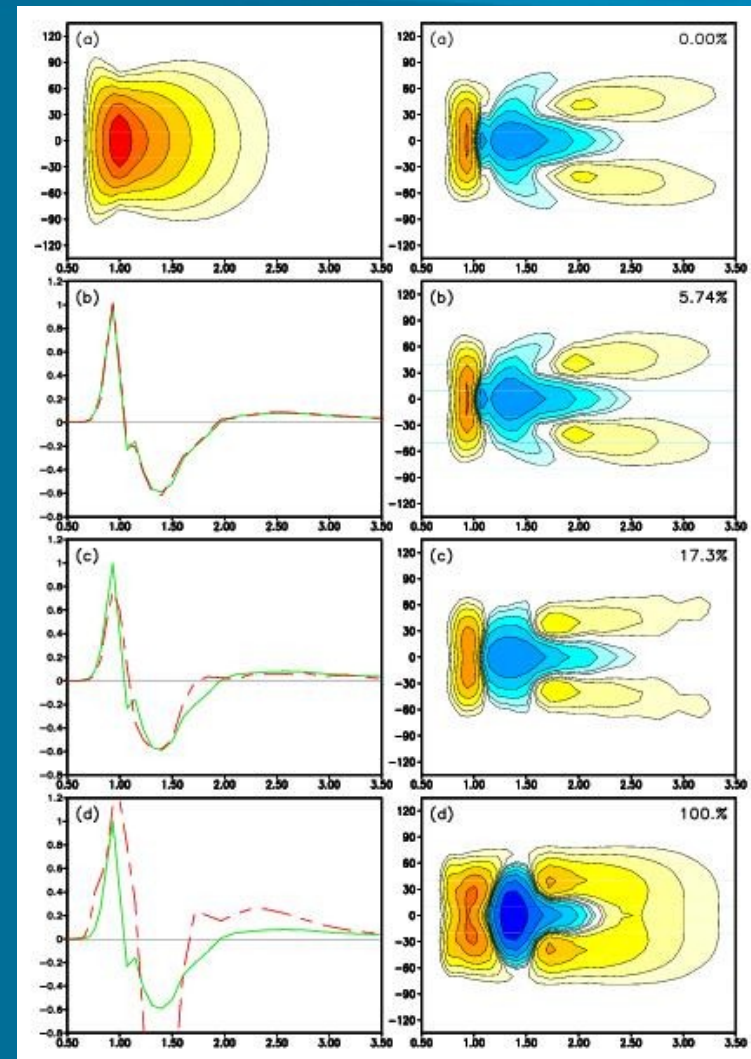
- Success of interactions from DIA with respect to wave heights is misleading,
- Near identical wave heights from various interaction approaches can lead large differences in spectral shape.
- WAVEWATCH III present approach has issues with location of spectral peak.



Physics (quadruplet interactions)



- Computation of interactions for test spectra show that much better results can be obtained than with DIA.
- Better interactions do not necessarily result in better model integration.
VDIA is unstable !
- Holistic optimization based on full model integration is needed.
- Shallow water scaling is needed.



Exact
(WRT)

MDIA
 $N = 4$

VDIA
 $N = 1$

DIA

Tolman, *Ocean Mod.*, 2004(6)



In most early 3G models, this is the tuning term
and not much more

- Original in WAM “weak in the mean” (= linear) only.
 - WAVEWATCH III default (Tolman and Chalikov 1996) :
 - Peak frequency based on wind sea only.
 - Linear at low frequency, local nonlinear at high frequency.
 - Based on wind stress (shallow, currents ?).
 - Since then, much progress from Australia
 - Saturation based on wave breaking in groups,
 - Explicit estimation of breaking occurrence and intensity.
- Can only be expected to come to full fruition with a better parameterization of S_{nl} .



Recently, an additional deep-water source term has become more prevalent.

$$\sum S(\mathbf{i}) = S_{in}(\mathbf{i}) + S_{nl}(\mathbf{i}) + S_{ds}(\mathbf{i}) + S_{sd}(\mathbf{i}) + \dots$$

- Swell dissipation is essential to be included in operational models.
- Dissipation with time scale of days to weeks is small compared to whitecapping, but notable over life of swell.
In WAVEWATCH III as negative wind input needed to eliminate model biases in tropical Pacific.
Inconclusive from 1960;s “waves across the Pacific” experiment.
Recent SAR observations from Ardhuin *et al.*



- So far we have looked at deep water physics only.
- Shallow water physics are additional, and can only work properly if the deep water part is taken care off,

but,

- Most people that interact with waves live on the coast, and therefore are affected by the shallow water effects ...

Note that :

- Refraction and shoaling are part of linear propagation and are relatively well understood:



- Wave-bottom interactions occur in intermediate water depths (outer shelf up to coast), and have been long been investigated (Shemdin et al, 1978, review paper)

Bottom friction most generally addressed:

Simple linear JONSWAP expression,
or drag-law style (Madsen et al.).

Interaction through oscillatory turbulent boundary layer:

- No impact of mean currents other than kinematics.
- Rough turbulent boundary layer means that friction is determined by physical bottom roughness (k_N or z_0).
- wave-induced ripples change roughness by orders of magnitude, but require sub-grid approach.



Friction cont'd :

- Don'ts for bottom friction:
 - Interaction goes through turbulence, therefore do not add wave and mean current velocities.
 - A friction factor associated with bottom friction varies with flow conditions. Pre-describing a friction factor rather than a roughness over-estimated nonlinearity.
- Physically most sound approaches :
 - JONSWAP linear term is based on observations, but constant does not appear universal.
 - Nonlinear dissipation based on physical bottom roughness
 - Many different bottom types, including moving sediments.



Other bottom interaction mechanisms:

- Percolation:
Similar to JONSWAP linear friction term.
- Wave-mud interactions:
Several different models.
May influence dispersion relation of spectral components.
Possible interplay with three-wave interactions ?
Hot item in US with New Orleans problems.



Shallow water (depth-induced) breaking is dominant dissipation mechanism in surf zone.

- Orders of magnitude more intense than whitecapping.
- All parameterizations have root in Battjes and Janssen (1978) parameterization.
- Key issues:
 - Behavior on different slopes (including flat bottom).
 - Integration with whitecapping.
 - Phase resolving versus phase averaging.

- Note: This source term is the “safety valve” for limiting wave heights in shallow water.
- Note: This source term is key in coupling wave and surge / inundation models.



Triad resonant interactions can only occur in extremely shallow water.

- Shifting energy to higher frequencies.
- Avoid unrealistic long periods on the beach.
- Simple parameterizations available in, e.g., SWAN.
- Key issues:

Work reasonably well on beaches, but not on extended shallow areas.

Is “no parameterization” better than poor parameterization?

Phase resolving versus phase averaging; is this a local or cumulative process?



Other physics parameterizations have been suggested

- Effects of rain on wave.
 - Effecting capillary waves and therefore remote sensing.
- Scattering of waves due to interaction with smaller scale bottom features.
- Is there a need for a new source term for wave energy dissipation in case of wave blocking?
- Advanced wave-ice interactions.
 - Models are available with a complexity and structure comparable to S_{nl} .
- I would expect to see effect of oil and other slicks on waves to be modeled explicitly in the near future.



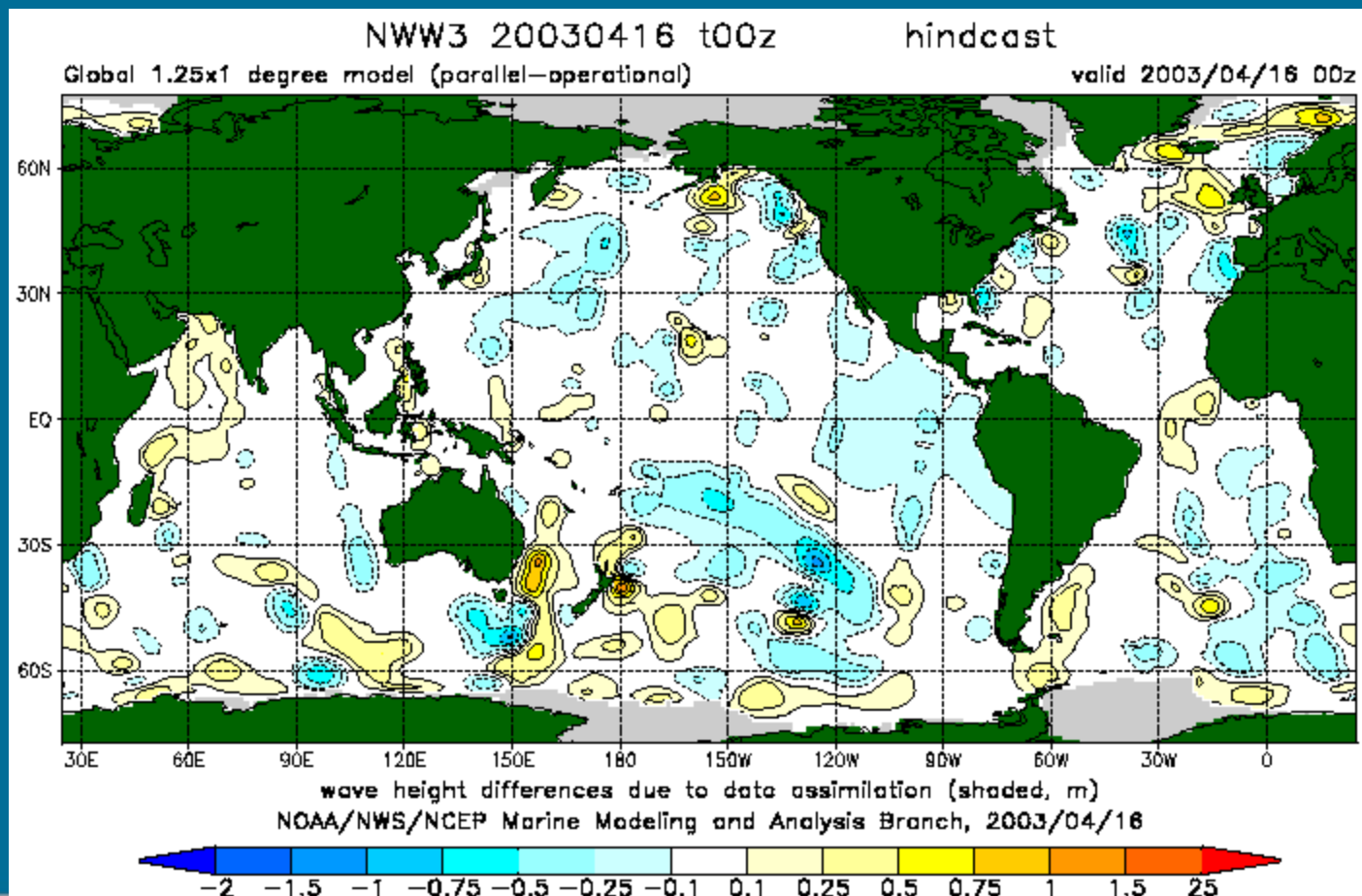
This is not a weather model !!!!

- Wind waves represent a forced and damped problem.
This is not an initial value problem like a weather or ocean model.
- Therefore, a very good wave model can be build without any data assimilation.
Weather and ocean models are critically dependent upon initial conditions and therefore on data assimilation.
- Due to the forced and damped nature, retention time of data into a wave model is limited by definition:
Typically 12-36 hours in terms of impact on wave height in typical operational models.
May be much smaller if a coastal model uses coastal buoy data.
Up to weeks for Pacific swell.



There is not enough data.

- There is no data-dominated analysis.
- Quality of analysis is by definition misleading.





Is there a need for DA in wave models?

- Yes, our operational customer needs the best possible model, and DA will improve short-term forecasts.
- There are political reasons to do DA.
 - Establish real-time operational use of data sources.
 - This justifies existence of data sources.
 - Data sources are critical for off-line development and quality assessment of operational models.
 - Off-line use does not carry weight with administrators like real-time use does.



Historical approaches to DA:

- Green's function approach by Hasselmann et al:
 - Use inverse model to estimate errors in wave generation.
 - Correct model forcing to minimize wave generation errors.
 - Re-run model with improved forcing.
- Features:
 - Expensive, therefore going back only hours, not days to improve forcing.
 - Requires enough data to constrain error in full forcing field.
 - Assumes perfect wave model.
- Not used in any operational model.



History cont'ed:

- Updating of forcing by describing wind fields as splines (De Valk et al.)
 - Similar in concept to previous page, but cheaper.
 - Not used in any operational model.
-
- Both approaches have advantages that wind sea is modified consistently with the wind field, and, if corrections of forcing are used in forecast too, will remain in the model system. Swell is perturbed automatically.



History cont'ed:

- Analysis of wave height based on altimeter and buoy data, using OI or 2DVar methods.
- Features:
 - Analyze wave height, but need to update spectrum. This is by definition a subjective process.
 - Using techniques from other fields (SST and others), without acknowledging wave physics.
 - Original work at KNMI (De Las Heras et al): DA cannot correct for errors in numerical propagation scheme.
 - Contrary to assumption of perfect model in previous methods.
 - Economical.



OI / 2DVar cont'ed:

- Can be augmented with assimilation of swell observation (spectral buoy data, SAR data).
- More advanced methods using wave physics are slowly coming to fruition (Greenlade et al.)
- Can potentially be used in combination with ensemble information.

Ensemble Kalman Filtering.

Hybrid 2DVar-Ensemble methods,

- Used operationally world wide.



Ensemble of wave models are produced in several institutes:

- Only NCEP ensemble shows reasonable spread of wave heights.
 - Mostly a statistical issue, not wave modeling,
 - But wave physics are essential to understand and design a proper wave ensemble system
-
- Will be discussed in some more detail in lecture wwvs 2.3



- 3G wind wave models are well established, and given a reasonable wind forcing, can be of very high quality.
 - This does not mean that all issues are solved.
Using waves in coupled models, remote sensing and ecosystems requires accurate spectra, not just wave height.
 - In the following slides, some issues, ongoing work and needed work is identified. Again making a distinction between
Numerics (and technology),
Physics, and
Assimilation.
- A lot of this represents Hendrik's personal beliefs and views!



Numerics and technology

- Alternate grid approaches (curvilinear and unstructured) are being developed:

SWAN (UNSWAN).

WAVEWATCH III. I believe in a mosaic approach where structured (including curvilinear) and unstructured grids are used side-by-side.

Structured for global ocean.

Curvilinear over North Pole.

Unstructured on the coast.

Elements available in experimental versions of the model, but still need to be integrated into the mosaic.

Erick Rogers and co. (NRL).



N&T cont'd

- The eternal issue of accurate yet economical advection.
- With alternate grid approaches for coastal applications, a quasi-stationary approach for high-resolution grids needs to be included in WAVEWATCH III.
 - Iterative elliptical solution is not that different from iterative hyperbolic solution.
- The GSE is now alleviated, but should be solved.
- Blocking:
 - Accuracy of numerics has not been assessed adequately.



N&T cont'd

- Source term integration:

Dynamic integration technique has served us well in WAVEWATCH III.

Recent question marks posed by Tamura et al. (JPO, 2008).

Build-in conflict of solution techniques.

Accuracy of integration near spectral peak (appears to be issue on Tamura et al.), versus,

Stable equilibrium solution in spectral tail (Hargreaves and Annan, 1998, 2001).



Physics:

- Do we need nonlinear propagation in the models?
Willebrand (1974) nonlinear shallow water propagation.
Nonlinear propagation at wave blocking (Chawla and Kirby).

Extension to 3G models.

- The more elementary question is: does the sine-base random phase model suffice?

Stokes based descriptions (Chalikov, Janssen).

Is phase information needed, particularly in depth-limited water?

Evolution to 4G models.



Physics cont'ed:

- The 3(4) deep water source terms seem to work reasonably well in most models, however, a structural re-evaluation is due.

NOPP project starting 2010 to upgrade these source terms to state-of-the-art.

See previous source term slides.

Note default in WAVEWATCH III is from 1995!

Adding the swell dissipation as an additional process.

- Wind input and/or dissipation ?



Physics cont'ed:

- The description of triad interactions is highly unsatisfactorily in 3G models.
 - Integrate with quadruplets (NOPP funded).
 - Phase averaged versus phase resolving.
 - Can this be done purely local ? and
 - Can this be separated from breaking ?
- Breaking is described by two or even three processes: (whitecapping, surf breaking, blocking dissipation).
 - Separate description prone to “double counting”.
 - Integration in single process desirable.
 - Assuming that all breaking represent similar physical processes.



Assimilation:

- Assimilation has not gotten the attention it deserves, but for good reasons.
- OI / 2DVar approaches represent an economical basis for doing assimilation.
- Possible research and improvement topics:
 - Add wave physics to data assimilation:
 - Use ensemble information.
 - Use partitioned wave field information.
 - Wave height versus spectrum.
 - Use all data (particularly spectral).
 - Use situation dependent error information (use physics).

The end



End of lecture wwvs 2.1