The Time Dependent Ray Method for Calculation of Wave Transformation on Water of Varying Depth and Current [1]

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Abstract

This paper focuses on an aspect of the numerical calculation of wave refraction using the ray tracing or the semi-Lagrangian approach. The conventional ray method for steady state depth and current fields is extended to the unsteady situation. It is shown that a factor which characterizes the unsteadiness of the medium can be included explicitly in the calculation of the wave energy amplification. The contribution associated with this factor on wave energy is $c_o{}^2/(\mathbf{C_g} + \mathbf{U})^2$, where $\mathbf{C_g}$ and $\mathbf{U}$ are the wave group velocity and current velocity, respectively, and c_o is the phase velocity of the wave-like fluctuation of currents and depth (e.g., caused by tides) in the direction of the ray path, i.e., in the direction of wave energy propagation.

Introduction

Efforts to include the effects of wave refraction due to spatially and/or temporally varied current and water depth in a numerical wave prediction model have been made by a number of researchers (see, e.g., Collins, 1972; Cavaleri and Rizzoli, 1981; Chen and Wang, 1983; Tolman, 1989,1991; Hubbert and Wolf, 1991). Numerical schemes for wave propagation and refraction calculations employed by these researchers can be conveniently classified into two types of approach. The Eulerian approach applies a finite difference scheme to obtain solutions at each grid point over the area of interest simultaneously. The Lagrangian approach uses the ray tracing technique to derive solutions at each specified point independently.

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The advantage of the Eulerian approach is that it provides a synoptic view of the wave pattern involving all frequency and directional components over the entire area and allows addition of the various source and sink terms including nonlinear wave-wave interactions. However, finite difference schemes for the convective transport equation have a common problem - the solution always involves unavoidable numerical errors (numerical dispersion and diffusion). As a result, the physical significance of their solutions is obscured.

In contrast, the Lagrangian approach (or what comes to the same thing: the method of characteristics or the ray method) follows a wave packet along the wave ray, thus avoiding the troublesome convective term and providing a strong physical realization of wave propagation. It provides a true path and time for any choice of wave parameters arriving at a specific point. Consequently, the loss of spatial generality is compensated by an increase in temporal accuracy.

There are limitations to this approach. It does not provide the synoptic view of the wave field over the entire area of interest at each computational time step. It is incapable of modeling the effects of source terms which involve all spectral components at each time step, such as nonlinear wave-wave interactions and white capping. It is also difficult to specify quantitatively the energy amplification factor in the vicinity of ray crossings and caustics which may occur if waves propagate over a complicated bottom topography or current field.

An early effort to combine the strengths and eliminate the shortcomings of both the finite difference and ray tracing techniques has been made by Barnett et al. (1969, also see, Allender et al.,1985). They developed a scheme which constructed a set of parallel rays over the ocean for each specified wave direction. Along each ray, the distance between ray points is determined according to the group velocity of the given frequency and time interval. The wave energy, considered as particles, hop from ray point to ray point at appropriate multiples of the time step. The wind field is provided on a net of grid points defined over the area of interest. At each given time step, the wave spectrum at any grid point is accumulated from the nearest ray point for each frequency and direction. After the spectrum is constructed, the processes of wave growth, dissipation and wave-wave energy transfer are executed. The spectral components are then re-distributed back to the appropriate ray points. Their approach can be applied only in deep water without currents and under steady state conditions since it assumes the constancy of wave direction along the rays.

It is of interest to note that the so-called semi-Lagrangian technique, which has gained widespread recognition as an efficient way to integrate the primitive equations of numerical weather prediction, also has been applied to the numerical prediction of ocean waves in recent years. Basically, it is an extension of the classical treatment of the refraction problem at a given site using the backward ray tracing technique (Dorrestein, 1960). With this approach, each grid point of a grid mesh covered the entire area of concern is considered to be a target point to receive the wave energy of all spectral components in a specified time step from various source locations. The source location of each component (the departure point) is determined by the method of ray back-tracing for the given time step. The localized physical processes such as wave growth by wind, dissipation due to whitecapping and wave-wave energy transfer are calculated at each grid point. The calculated spectral energy densities at all grid points serve to provide information for deriving the spectral energy density at each source location. Since it is impossible to have the departure point always coincide with a certain grid point, an appropriate interpolation procedure is performed to obtain the energy content at the departure point based on the values at neighboring grid points. A feasibility study of this approach along with different methods for spatial interpolation as a part of this system, has been studied by Ryabinin (1991).

The scheme is potentially attractive because of its high accuracy and efficiency in computation by allowing large time intervals and low spectral resolutions. In shallow water, the energy dissipation due to bottom friction can be computed along the rays. The effect of sub-grid irregularity in bottom topography and/or surface currents on the wave energy amplification can be calculated explicitly, and the problem of crossing rays and caustics can be avoided as the wave propagation distance is relatively short in a given time step. In addition, the underlying grid mesh can be arranged irregularly depending on the accuracy of the input data and the requirements for wave information in the area of interest.

A problem in applying this semi-Lagrangian approach, however, remains to be solved. In connection with the study of wave-current interaction problem in the southern North Sea, Tolman (1990) pointed out that for the large scale continental shelf unsteadiness of current and water depth induced by tides must be considered if the time scale of variation in current and depth (typically 12 hours) is not large compared to the travel time of the waves through the area of interest. The wave energy amplification factor derived from the conventional ray method is derived based on the assumption of steady state water depth and flow conditions. Therefore the effect of unsteadiness in depth and currents cannot be evaluated. In this paper, the time dependent ray method is derived to express explicitly factors involved in calculating the change of wave spectrum due to the existence of unsteady and irrelegular depth and current fields.

Basic Equations

The change of wave field due to the presence of varying currents and bathymetry can be specified based on the dynamic conservation of wave action and the kinematic conservation of wave number or wave crests along characteristic curves or rays (Bretherton and Garrett, 1969; Phillips, 1977). The path of a ray is determined by simultaneous solution of the following set of equations:

$$\frac{dx_j}{dt} = \frac{\partial \omega}{\partial k_j} = c_{g_j} + u_j, \qquad (1)$$