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A TECHNIQUE FOR VECTOR CORRELATION AND ITS APPLICATION TO MARINE SURFACE WINDS

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1. INTRODUCTION

The problem of correlating vector quantities has been of interest to meteorologists for at least the past 75 years (e.g., Sverdrup, 1917; Durst 1957; Court, 1958; Breckling, 1989). However, it appears that a completely satisfactory definition for vector correlation has yet to emerge. Crosby et al. (1992) proposed a definition for vector correlation which arose outside the meteorological community, originating with Hooper (1959) and later expanded upon by Jupp and Mardia (1980). We apply the results of Crosby et al. to the problem of comparing marine surface winds for two different situations. In the first situation, the above definition for vector correlation is applied to marine surface winds from buoys at two locations in the NW Atlantic approximately 700km apart; in the second, we compare marine surface winds derived from NMC's Global Data Assimilation System with those acquired from NDBC buoys located primarily in U.S. open coastal waters and in the Gulf of Alaska. The data selected in the first case are time series, and as such, allow us to examine the time variation in vector correlation over the length of record. In the second case, the observed and analyzed data were simply grouped by month, permitting intermonthly and seasonal comparisons.

First, we present a brief review of the theory and a description of the properties associated with the definition of vector correlation originally given by Hooper. Then we apply the technique to marine surface winds in two different situations. We summarize our results and comment on the technique in the final section of the paper.

2. THEORETICAL BACKGROUND AND PROPERTIES

2.1 Theory

Given the two-component vectors $\vec{W}_1 = u_1 i + v_1 j$ and $\vec{W}_2 = u_2 i + v_2 j$ in Cartesian coordinates, we can express the covariance matrix for $\vec{W}_1$ and $\vec{W}_2$ as

$$\sum_{\vec{W}} = \begin{bmatrix} \sum_{11} & \sum_{12} \\ \sum_{21} & \sum_{22} \end{bmatrix} \quad (1)$$

where the Σ_{ij} submatrices are given by

$$\sum_{11} = \begin{bmatrix} \sigma(u_1, u_1) & \sigma(u_1, v_1) \\ \sigma(v_1, u_1) & \sigma(v_1, v_1) \end{bmatrix} \quad (1a)$$

$$\sum_{12} = \begin{bmatrix} \sigma(u_1, u_2) & \sigma(u_1, v_2) \\ \sigma(v_1, u_2) & \sigma(v_1, v_2) \end{bmatrix} \quad (1b)$$

$$\sum_{21} = \begin{bmatrix} \sigma(u_2, u_1) & \sigma(u_2, v_1) \\ \sigma(v_2, u_1) & \sigma(v_2, v_1) \end{bmatrix} \quad (1c)$$

$$\sum_{22} = \begin{bmatrix} \sigma(u_2, u_2) & \sigma(u_2, v_2) \\ \sigma(v_2, u_2) & \sigma(v_2, v_2) \end{bmatrix} \quad (1d)$$

In Eq. (1), the σs are the variances or the covariances of the u and v components. The vector correlation between $\vec{W}_1$ and $\vec{W}_2$ is then defined as

$$\rho_v^2 = TR\left(\left(\sum_{11}\right)^{-1}\sum_{12}\left(\sum_{22}\right)^{-1}\sum_{21}\right) \quad (2)$$

where ρ_v^2 is the square of the vector correlation coefficient, ρ_v, and TR represents the trace of the products of the Σ_{ij} submatrices (Jupp and Mardia, 1980). Eq. (2) can be expanded in algebraic form to yield

$$\rho_v^2 = f/g, \quad (3)$$

where

$$\begin{aligned} f = {} & \sigma(u_1,u_1)(\sigma(u_2,u_2)(\sigma(v_1,v_2))^2 + \sigma(v_2,v_2)(\sigma(v_1,u_2))^2) + \\ & \sigma(v_1,v_1)(\sigma(u_2,u_2)(\sigma(u_1,v_2))^2 + \sigma(v_2,v_2)(\sigma(u_1,u_2))^2) + \\ & 2(\sigma(u_1,v_1)\sigma(u_1,v_2)\sigma(v_1,u_2)\sigma(u_2,v_2)) + \\ & 2(\sigma(u_1,v_1)\sigma(u_1,u_2)\sigma(v_1,v_2)\sigma(u_2,v_2)) - \\ & 2(\sigma(u_1,u_1)\sigma(v_1,u_2)\sigma(v_1,v_2)\sigma(u_2,v_2)) - \\ & 2(\sigma(v_1,v_1)\sigma(u_1,u_2)\sigma(u_1,v_2)\sigma(u_2,v_2)) - \\ & 2(\sigma(u_2,u_2)\sigma(u_1,v_1)\sigma(u_1,v_2)\sigma(v_1,v_2)) - \\ & 2(\sigma(v_2,v_2)\sigma(u_1,v_1)\sigma(u_1,u_2)\sigma(v_1,u_2)) \end{aligned}$$

and

$$g = [\sigma(u_1,u_1)\sigma(v_1,v_1)-(\sigma(u_1,v_1))^2][\sigma(u_2,u_2)\sigma(v_2,v_2)-(\sigma(u_2,v_2))^2].$$

This version of Eq. (2) may be more convenient for computational purposes. Because W is two-dimensional, Eq. (2) yields values of ρ_v^2 which vary between 0.0 (no correlation) and 2.0 (perfect correlation).[1] For convenience, we have calculated, and quote, values of ρ_v^2 (vice $\sqrt{\rho_v^2}$) throughout the study.

2.2 Properties

The definition for vector correlation given by Eq. (2) has the following properties, where we have replaced the population parameter, ρ_v^2, by the corresponding sample parameter, r_v^2.

(i) r_v^2 is symmetric in the sense that

$$r^2_{\vec{W}_1|\vec{W}_2} = r^2_{\vec{W}_2|\vec{W}_1}$$

(ii) r_v^2 is independent of coordinate transformations.

(iii) r_v^2 is equal to 2.0, for two-dimensional vectors, if $\vec{W}_1$ and $\vec{W}_2$ are completely dependent.

(iv) If $\vec{W}_1$ and $\vec{W}_2$ are independent, then r_v^2 will approach 0.0 as the sample size increases without limit. For $\vec{W}_1$ and $\vec{W}_2$ independent, nr_v^2 is asymptotically distributed as chi-square, where n is the sample size for which r_v^2 is calculated. For the two-dimensional case, the chi-square distribution has four degrees-of-freedom.

(v) For the one-dimensional (scalar) case, r_v^2 simplifies to the square of the Pearson product-moment correlation coefficient.

(vi) The vector correlation (squared), r_v^2, is equal to the sum of the squared first (r_1^2) and second (r_2^2) canonical correlations (Crosby et al., 1992), $r_v^2 = r_1^2 + r_2^2$.

3. APPLICATIONS

3.1 Marine Surface Winds at Two Locations in the NW Atlantic

In the first situation, we calculate vector correlations between surface winds at two locations in the NW Atlantic. The wind observations were acquired by NDBC environmental data buoys located at 40.5°N, 69.5°W (buoy number 44008) and at 34.9°N, 72.9°W (buoy number 41001). These buoys, whose locations are shown in Figure 1, are approximately 700km apart, close enough so that synoptic-scale disturbances that typically pass through the region will, in most cases, influence the winds at both sites. An expected winter storm track for this region has been included (Klein, 1957). As winter low-pressure systems leave the east coast of the U.S., they often deepen over the Gulf Stream and expand as they propagate to the NE. Thus, the winds at both buoys are expected to be strongly influenced by the passage of these low pressure systems which pass through the area during the winter months. The observations, taken hourly, extend from 1 December 1987 to 4 February 1988, a period of 65 days. The stick diagram shown in the upper two panels of Figure 2 depict the time series of wind vectors at each location.

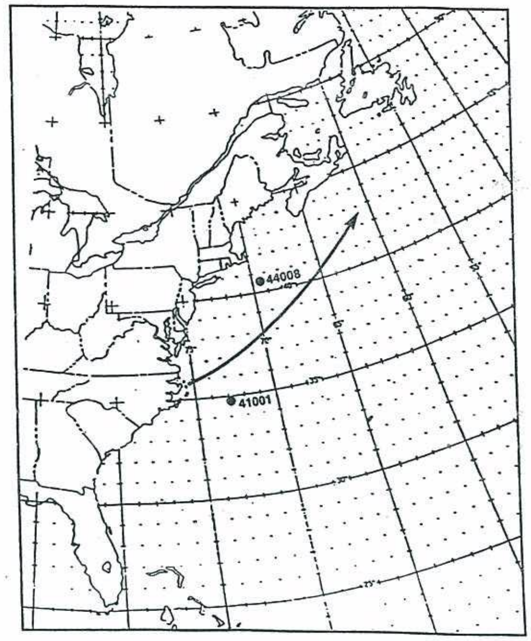

FIGURE 1. Locations of the two NDBC environmental data buoys from which time-series surface winds were extracted. Period covers 1 December, 1987 to 4 February, 1988. A typical winter storm track has been included (Klein, 1957).

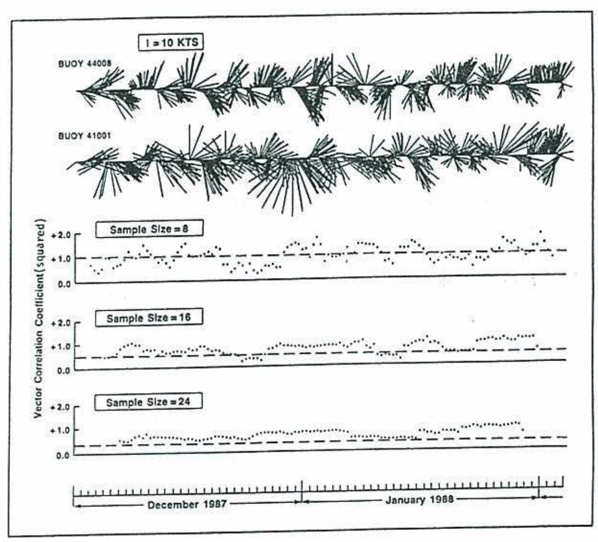

Figure 2. Wind vector sequences for NDBC buoys 44008 (top panel) and 41001 (next-to-top-panel), and the corresponding vector correlations for sample sizes of 8, 16, and 24 (lower panels). The horizontal dashed lines in the lower three panels indicate the 5% level of significance.

[1]This definition, of course, can be scaled to vary between 0.0 and 1.0, if so desired.