

NASA Technical Memorandum 87812

Tidal Estimation in the
Atlantic and Indian Oceans,
 $3^{\circ} \times 3^{\circ}$ Solution

Braulio V. Sanchez, Desiraju B. Rao,
and Stephen D. Steenrod

APRIL 1987

NASA

Tidal Estimation in the
Atlantic and Indian Oceans,
 $3^{\circ} \times 3^{\circ}$ Solution

Braulio V. Sanchez
*Goddard Space Flight Center
Greenbelt, Maryland*

Desiraju B. Rao
*National Oceanic and Atmospheric Administration
National Meteorological Center
Rockville, Maryland*

Stephen D. Steenrod
*Applied Research Corporation
Landover, Maryland*



National Aeronautics
and Space Administration

Scientific and Technical
Information Branch

I. INTRODUCTION

The extraction of ocean tidal components from the analysis of satellite altimetry data could become an alternative and complementary method to the numerical integration of the Laplace tidal equations.

Mazzega (1985) created a global model of the M2 tide using twenty-four days of SEASAT altimetry data; the solution was obtained by means of surface spherical harmonics; the results were qualitatively realistic. No hydrodynamic equations were used.

Woodworth, P.L. and Cartwright, D.E. (1986) have extracted the M2 ocean tide from SEASAT altimetry data. They used three complementary methods. The first method provides point measurements of the tide at the crossovers of the SEASAT repeat orbit ground track; it was applied in the tropical ocean areas. The other two methods involve the spatial expansion of M2 in terms of either surface spherical harmonics or Platzman normal modes of the world ocean. The results reproduce many features of the tide represented in recent tidal models.

The purpose of this investigation is to develop an estimation technique which will serve to extrapolate tidal amplitudes and phases over entire ocean basins using existing gauge data and the precise altimetric measurements which are now beginning to be provided by satellite oceanography. The applicability of the extrapolating technique was tested in the Lake Superior basin by Sanchez, Rao and Wolfson (1985) and in the Atlantic-Indian Oceans (using a $6^\circ \times 6^\circ$ degree) by Sanchez, Rao and Steenrod (1986 a, b).

The method to be used in this investigation requires several distinct steps. First it is necessary to determine numerically the stream function and velocity potential orthogonal functions (The Stokes/Helmholtz Potentials) which span the space of the basin under consideration. These space functions are then used in the Laplace tidal equations to determine the homogeneous solution (normal modes) and the forced solution. The latter is obtained by adding the astronomical forcing function modified to include solid-earth tides.

The velocity potential eigenfunctions obtained as a first step are used also to extrapolate the surface height field over the entire space domain of the given basin and this approach will constitute a distinct and integral part of the investigation.

The theoretical foundation is Proudman's theory (1918) as formulated by Rao (1966). The theory provides the formalism for calculation of the gravitational (first class) normal modes and the rotational (second class or Rossby waves) normal models of irregularly shaped basins with realistic bathymetry.

The method requires the solution of two elliptic partial differential equations with second order operators which are simpler than the tidal operator. The boundary conditions correspond to vanishing of the stream function and normal derivative of the velocity potential. The elliptic operators are represented numerically in finite difference form, the grid used is a Richardson lattice which preserves self-adjointness. The solutions yield the velocity and surface height fields in terms of orthogonal functions with time-dependent coefficients. These functions are then substituted into the Laplace's tidal equations: if the homogeneous equations are used one obtains the normal modes; if the forcing terms are included then the forced solution is obtained. In both cases the solution is obtained numerically. The surface height field is only dependent on the velocity potential orthogonal functions. The expansion coefficients of these functions can be estimated in a least-square sense from available selected tidal measurements.

2. BASIC EQUATIONS

Free Solutions

The method of approach was developed originally by Proudman (1918) using a Lagrangian approach. It was reformulated by Rao (1966) from the Eulerian point of view. The basic ideas of the method presented below follow Rao's line of development. The basic equations are the linearized shallow water equations on a rotating plane:

$$\begin{aligned} \frac{\partial \vec{M}}{\partial t} - f[\vec{M}] &= -g \bar{H} h \nabla \eta \\ \frac{\partial \eta}{\partial t} + \nabla \cdot \vec{M} &= 0 \end{aligned} \quad (2.1)$$

where

$$\begin{aligned} \vec{M} &\equiv H \vec{V} \equiv (M, N) \\ \vec{V} &\equiv (u, v) \\ f &\equiv 2 \omega \sin \theta \\ h(x, y) &\equiv H(x, y) / \bar{H}. \end{aligned}$$

$H(x, y)$ is the variable depth of the fluid in equilibrium and \bar{H} is some constant scaling depth, f is the Coriolis parameter, \vec{V} is the horizontal velocity vector, η the fluctuation of the free surface. g is the apparent gravitational acceleration and ∇ is the horizontal gradient operator. $[\]$ denotes rotation of the vector through a right angle in the clockwise direction of the horizontal plane, i.e. $[\] = -(\vec{k} \times \nabla)$, \vec{k} being a unit vector vertical to the horizontal plane.

The appropriate boundary conditions to be adjoined to equations (2.1) are

$$\vec{M} \cdot \vec{n} = 0 \quad (2.2)$$

on the coast, where \vec{n} is the unit normal to the coastline.

The transport vector \vec{M} may be partitioned as follows:

$$\vec{M} = \vec{M}^\phi + \vec{M}^\psi \quad (2.3)$$

where

$$\begin{aligned} \vec{M}^\phi &= -h \nabla \phi \\ \vec{M}^\psi &= -[\nabla \psi] \end{aligned} \quad (2.4)$$

\vec{M}^ψ is the solenoidal part of \vec{M} while $h^{-1} \vec{M}^\phi$ is the irrotational part since

$$\begin{aligned} \nabla \cdot [h^{-1} \vec{M}^\phi] &= 0 \\ \nabla \cdot \vec{M}^\psi &= 0 \end{aligned}$$

To complete the determination of ϕ and ψ , it is necessary to specify the boundary conditions $\vec{M}^\phi \cdot \vec{n} = 0$ and $\vec{M}^\psi \cdot \vec{n} = 0$ to ensure that Eq. (2.2) is satisfied. In terms of ϕ and ψ the conditions then are

$$h \frac{\partial \phi}{\partial n} = 0 \quad (2.5)$$