

NASA Technical Memorandum 87799

A Method of Calculating the  
Total Flow From a Given  
Sea Surface Topography

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APRIL 1987

**NASA**

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# A Method of Calculating the Total Flow From a Given Sea Surface Topography

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## INTRODUCTION

Altimetric measurements from a satellite yield the sea surface topography on a global basis. Such a sea surface topography represents the sum of (i) the marine geoid, which is the shape the ocean surface assumes when at rest, (ii) the departure of the ocean surface from the geoid due to the near surface ocean circulations, and (iii) a variety of errors related to the determination of satellite orbit, tides, etc. See Wunsch and Gaposchkin (1980) and report of the TOPEX Science Working Group (1981) for a review of the errors associated with satellite altimetric measurements. If the errors due to source (iii) are eliminated and if one knows the shape of the geoid, in principle, we have the residual sea surface topography associated with dynamics of the ocean circulation. Since the surface elevation is a simple scalar variable and reflects oceanic processes occurring at depth, satellite altimetry offers the potential to study global ocean circulation if it can be measured with the necessary accuracy – particularly when treated together with other available hydrographic data. See, for example, the analysis of the Seasat altimetric residuals by Tai and Wunsch (1983) and Tai (1983).

The primary advantage of being able to measure the sea surface topography is the fact that one can directly compute the surface pressure gradients and the associated geostrophic currents. These currents would then be the reference velocities for the integration of the observed density field via the “thermal wind” equation to provide absolute currents at depth. Hence the altimetric measurements of the sea surface eliminate the necessity of invoking the existence of an arbitrary level of no motion which can only provide the relative currents. However, if certain assumptions are invoked and a conceptual model of ocean circulation dynamics is adapted, it is possible to infer more than the “near surface geostrophic currents” from the measured sea surface topography. Admittedly the ocean circulation is a complex result of forcing by the wind stresses and the atmospheric heating and cooling. The circulation consists of both barotropic and baroclinic components and exhibits spatial and temporal variabilities on a wide range of scales. Nevertheless, in this preliminary attempt, a working assumption is made that the circulation dynamics in the upper layers of the ocean are dominated by wind driven forcing much like in the Stommel (1948) study. When the upper layer of an ocean is spun up to a steady state under the influence of a wind stress and a linear bottom friction, the circulation dynamics represent a balance between the Coriolis, pressure gradient, wind stress, and bottom friction forces. Hence the observed surface topography represents a more complicated balance of forces than just geostrophy even in this simple conceptual dynamical framework.

We present a method here that, given a data field of sea surface topography, allows the recovery through an objective analysis procedure of the total flow field, which may be viewed as the sum of a geostrophic and an ageostrophic component if one so desires. However, since geostrophic considerations become progressively weak as one approaches the equator and the flows become more and more ageostrophic, we will simply refer to the flow derived from our calculations as the actual or total flow. Since no geostrophic considerations are imposed in the analytical development, the analysis procedure outlined here is not affected by the vanishing of the Coriolis parameter at the equator in calculating the currents from the surface pressure field.

The results presented deal with the steady circulation in an ideal rectangular basin on a beta plane. The theory is, however, also valid to study the transient circulation so long as the circulation is assumed to be non-divergent.

The theory is based on developing a spectrum of characteristic functions for the stream function field and the surface height field. Given a wind field these functions are then used to represent the forced solution for the velocity and surface pressure fields in which the expansion coefficients for both fields are required to be the same. Such a calculation serves a two-fold purpose. The first one is that the forced solution indicates which of the spectral components are most energetic so that these may be used in the objective analysis of the height field data. The second purpose is that when an objective analysis is performed on a limited set of surface height field data points extracted from the theoretical solution, this solution would serve the function of the “true” solution to validate the objective analysis procedure. The reason for using a limited