

Alleviating the Garden Sprinkler Effect in wind wave models [☆]

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Abstract

In ocean wave models, swell propagation at coarse spectral resolution leads to the disintegration of continuous swell fields into discrete swell fields. This process is known as the Garden Sprinkler Effect (GSE). An existing solution to the GSE consists of adding a diffusion tensor to the propagation equation. Although this diffusion method has been proven successful, it is prohibitively expensive for models with high spatial resolution. Two alternatives are presented here. The first is an averaging method. It shares characteristics with the diffusion method, but is much cheaper for high resolution models. The second method consists of adding divergence to the advection field. This divergence method is shown to be accurate for idealized conditions, and requires less tuning, but is still too expensive to replace the other methods in practical conditions. It is therefore suggested to replace the diffusion method with an averaging method in operational models, and to investigate the divergence method further. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Ocean wave models solve some form of the spectral energy or action balance equation, such as

$$\frac{DF}{Dt} = S, \quad (1)$$

where F is a wind wave spectrum, and S represents source terms for spectral wave energy due to the influence of wind, wave breaking ('whitecapping'), nonlinear interactions, and additional (mostly shallow-water) processes. In the spectral description used in Eq. (1), the spectrum F and

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source term S are functions of two parameters describing spectral space. Conventionally, these have been the spectral frequency f and direction θ . Although alternative descriptions have been used, the differences are immaterial for the present study. The spectrum is furthermore a slowly varying function of space \vec{x} and time t (compared to the length and period of individual waves)

$$F = F(f, \theta; \vec{x}, t), \quad S = S(f, \theta; \vec{x}, t). \quad (2)$$

In a conventional numerical wave model, spectral space (f, θ) and physical space \vec{x} are discretized, while the solution is propagated with discrete time steps Δt . The need to discretize in four dimensions makes it difficult to attain resolution and economy at the same time. In state-of-the-art wave models like WAM (WAMDIG, 1988; Komen et al., 1994), WAVEWATCH III (Tolman and Chalikov, 1996; Tolman, 1999), and SWAN (Booij et al., 1999; Ris et al., 1999), a typical spectral resolution is 24 directions with $\Delta\theta = 15^\circ$, and a logarithmic frequency distribution

$$f_{i+1} = \gamma f_i, \quad (3)$$

where i is the discrete grid counter in f -space, and typically $\gamma = 1.10$. Better resolutions have been used, but the parameterization of nonlinear interactions as used in these models (the Discrete Interaction Approximation, DIA, Hasselmann et al., 1985), appears best suited for such a resolution. Furthermore, this resolution appears sufficient to adequately describe (local) wave growth (e.g., Tolman, 1992). With this spectral resolution, spatial resolution appears to have been mostly dictated by economics, and to optimally use nominal resolutions of modelled wind fields employed to drive these models.

Whereas the above described common resolutions appear generally adequate in conditions of active wave generation, problems occur when active wave generation stops, and wind seas become swell. Such swells travel across the ocean, virtually without interacting with other wave groups (e.g., Snodgrass et al., 1966). Due to continuous dispersion of swell energy with different frequencies and directions, a swell field covers an increasingly large area. In numerical models, however, the spectral frequency and direction are not continuous but discretized. Discrete swell fields therefore travel in discrete directions with discrete speeds (i.e., discrete frequencies). If the spectral resolutions are inadequate, this will result in a spurious disintegration of the continuous swell field into discrete swell fields. This numerical problem is generally known as the Garden Sprinkler Effect (GSE).

The GSE and possible solutions to it have been discussed in great detail by Booij and Holthuijsen (1987, henceforth denoted as BH87). To illustrate the effects of the GSE, results of a test case similar to theirs are presented here. In this test, propagation in deep water in a Cartesian (x, y) space is considered. Eq. (1) then reduces to

$$\frac{\partial F}{\partial t} + c_x \frac{\partial F}{\partial x} + c_y \frac{\partial F}{\partial y} = 0, \quad (4)$$

where c_x and c_y are the group velocity components in the respective directions.

In an area of $4500 \times 3500 \text{ km}^2$, discretized with $\Delta x = \Delta y = 100 \text{ km}$, an initial wave field is placed at 500 km from the lower and left side (point $(0, 0)$ in all figures). The initial wave field has a significant wave height $H_s = 2.5 \text{ m}$. The mean direction $\theta_m = 30^\circ$, and the directional energy distribution is of the $\cos^2(\theta - \theta_m)$ type. The mean frequency $f_m = 0.1 \text{ Hz}$, and the distribution of energy in frequency space is Gaussian with a spread of 0.01 Hz. The distribution of wave energy in

physical space is also Gaussian with a spread of 150 km. The model is run for five days to propagate the swell field to the top and right edges of the area. Numerical results are obtained using the ULTIMATE QUICKEST (UQ) scheme of Leonard (1979, 1991). Calculations are performed with a test version of WAVEWATCH III. From the perspective of the present study, the only relevant differences between the test version of this model and its presently distributed version (Tolman, 1999) is the option to run the model on Cartesian instead of spherical spatial grids, and the inclusion of new GSE alleviation methods as presented in the present study.

Fig. 1(a) shows the initial conditions and final wave heights obtained with the conventional spectral resolution $\Delta\theta = 15^\circ$ and $\gamma = 1.10$. Fig. 1(b) shows the corresponding near-exact results as obtained with a highly increased resolution of $\Delta\theta = 2.5^\circ$, $\gamma = \sqrt{1.10}$ and $\Delta x = \Delta y = 25$ km. Whereas the ‘exact’ solution shows the expected continuous swell dispersion, the conventional model results show a clear GSE. Particularly, the discrete swell fields for individual discrete directions are obvious. In frequency space discrete swell fields travel at different speeds in the same direction. Because no clear separation of fields is obvious in the propagation direction, the frequency resolution $\gamma = 1.10$ is apparently sufficient for the propagation distances and spatial extent of the initial distribution considered here.

As already mentioned above, BH87 present solutions to the GSE. The obvious solution is to increase spectral, in particular directional, resolutions. The required resolution, however ($\Delta\theta \ll 5^\circ$), is not economically feasible in operational models. Alternatively, BH87 present a modified propagation equation:

$$\frac{\partial F}{\partial t} + \frac{\partial}{\partial x} \left[c_x F - D_{xx} \frac{\partial F}{\partial x} \right] + \frac{\partial}{\partial y} \left[c_y F - D_{yy} \frac{\partial F}{\partial y} \right] - 2D_{xy} \frac{\partial^2 F}{\partial x \partial y} = 0, \quad (5)$$

$$D_{xx} = D_{ss} \cos^2 \theta + D_{nn} \sin^2 \theta, \quad (6)$$

$$D_{yy} = D_{ss} \sin^2 \theta + D_{nn} \cos^2 \theta, \quad (7)$$

$$D_{xy} = (D_{ss} - D_{nn}) \cos \theta \sin \theta, \quad (8)$$

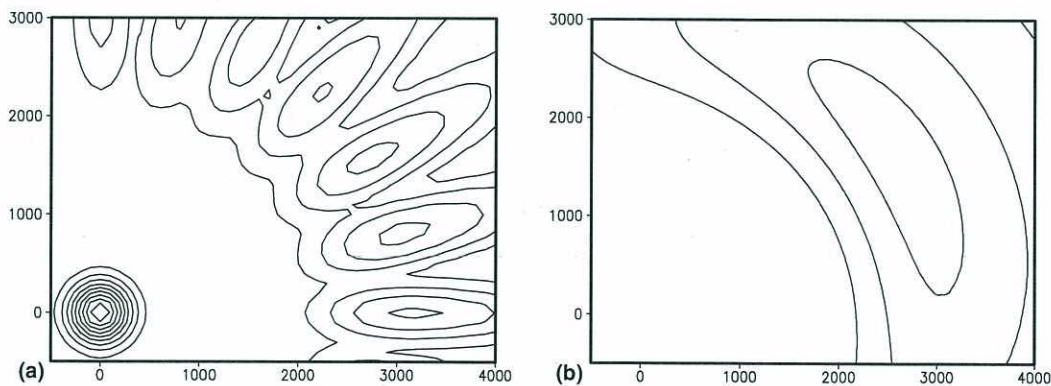


Fig. 1. Significant wave height (H_s) for simple GSE test as described in text. (a) Initial conditions near (0,0) with contours at 0.25 m intervals, and model results after 5 days with contours at 0.10 m intervals for conventional spectral resolution with $\Delta\theta = 15^\circ$ and $\gamma = 1.10$. (b) Near-exact results obtained with increased spatial and spectral resolution. Axes in km.