

LIMITERS IN THIRD-GENERATION WIND WAVE MODELS¹

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(Accepted November 2001)

Third-generation ocean wave models include a so-called limiter in the integration of the source terms to guarantee numerical stability at economical numerical time steps. The original limiter has previously been associated with the sensitivity of model results to the numerical time step. More recent limiters appear to remove this sensitivity by eliminating the numerical convergence from the resulting integration scheme. This is contrary to rudimentary numerical principles as well as the underlying philosophy of third-generation wave models. The present study investigates the effects of limiters and large model time steps using time-limited wave growth test. It is shown that the conventional limiter results in stable model results even if the numerical time step violates the time scales of wave growth. Contrary to common belief, its impact is not necessarily limited to the equilibrium range of the spectrum, and the limiter systematically *enhances* growth rates in the intermediate stages of wave growth. Particularly initial growth errors increase significantly with increasing maximum discrete spectral frequency f_{\max} . Relaxation of the limiter is shown to reduce initial growth errors, but does so at the expense of notable errors in the spectral shape. In the present paper the limiter was relaxed by introducing a new asymmetric limiter that retains full convergence. Initial results obtained with this limiter are similar to those of the advocated nonconvergent limiters. Although this limiter still needs rigorous testing and further development, its initial results suggest that there is no justification for using nonconvergent limiters.

Keywords: third-generation wave models; limiter; nonconvergence; parameterization

1. INTRODUCTION

Third-generation ocean wave models solve some form of the spectral energy or action balance equation, for instance

$$\frac{DF}{Dt} = S, \quad (1)$$

where F is the wind wave spectrum, and S represents source terms for spectral wave energy due the influence of wind, wave breaking ('whitecapping') nonlinear interactions and additional (mostly shallow-water) processes. In third-generation wave models, all sources on the right side of this equation are explicitly parameterized and accounted

¹OMB contribution Nr. 210.

for in the integration of the equation. The first operational third-generation wave model was the WAM model (WAMDIG 1998, Komen *et al.*, 1994), which solves Eq. (1) for the energy density spectrum $F(f, \theta)$ as a function of the spectral frequency f and the spectral direction θ . To achieve stable results at reasonable time steps, this model integrates the source terms in time using a semi-implicit scheme and a so-called ‘limiter’ that restricts the maximum change for each spectral bin per time step. Only recently this limiter and the argumentation behind it have been discussed in detail (Hersbach and Janssen 1999, henceforth as HJ99). In WAM cycles 1 through 3, the limiter (\mathcal{L}_0) is given as

$$\mathcal{L}_0 = 0.62 \times 10^{-6} g^2 f^{-5}, \quad (2)$$

$$-\mathcal{L}_0 < \Delta F_{\text{lim}} < \mathcal{L}_0, \quad (3)$$

where ΔF_{lim} is the discrete change of spectral energy density per time step after application of the limiter. The maximum change allowed (\mathcal{L}_0) corresponds to about 10% of the Phillips spectrum with the energy level according to Pierson and Moskowitz (1964) for fully developed seas¹. Furthermore, the integration is only performed up to a cut-off frequency f_c , above which a parametric tail corresponding to the Phillips spectrum is applied. This cut-off also helps to stabilize the integration, although arguments for applying such a cut-off appear rooted in physics rather than numerics. The cut-off frequency is usually set to 2.5 times the peak or mean frequency (see Komen *et al.*, 1994).

With the above integration method and limiter, WAM cycles 1 through 3 give stable results, but the results proved to be sensitive to the discrete time step Δt of the model (e.g., Tolman 1992, Fig. 1; HJ99 Section 1).

Tolman (1992, henceforth denoted as T92) removed the time step dependence from the integration by using the limiter to calculate the maximum allowed time step, and by dynamically adjusting the time step. When the time step is calculated separately for each spatial grid point, this method proves to be efficient for large-scale models², where conditions of active wave generation cover only a small part of the domain. For the active generation areas the time step then becomes significantly smaller, but for the remaining spatial grid points the time step can safely be chosen much larger than the customary 1200 s. In practice, this implies that the average source term integration time step for large-scale models significantly increases, making such models more economical to operate. This method is also advocated by Hargreaves and Annan (2001). Reservations voiced by Hersbach and Janssen (2001) are based on an erroneous representation of T92 (see Appendix).

The above solution is not applicable to the WAM model, because the design of this model requires that a single overall time step be used throughout the model. In WAM cycle 4, along with new physics parameterizations, a new formulation for the limiter was introduced to reduce the time-step dependence of the solution (see HJ99). This limiter is given as

$$\mathcal{L}_4 = \mathcal{L}_0 \frac{\Delta t}{\tau} \quad (4)$$

¹That is, in the mean wave direction, assuming a \cos^2 type directional distribution.

²See Appendix for relevant details of implementation.

$$-\mathcal{L}_4 < \Delta F_{\text{lim}} < \mathcal{L}_4, \quad (5)$$

where $\tau = 1200$ s is a normalization time step. Like this limiter, the discrete change of spectral density per time step ΔF approximately scales with the time step Δt . Thus the relative impact of this limiter is independent of the time step and will not disappear for $\Delta t \rightarrow 0$. This implies that the limiter becomes an integral part of the solution, and that a numerical scheme that includes this limiter no longer converges to the solution for the physics parameterizations of the model. In fact, a model based on this limiter may be expected to closely reproduce the previous solution for a time step $\Delta t = 1200$ s, independent of the actual time step Δt . Convergence of the integration of source terms and hence an independence of the numerical solution from the limiter requires that the impact of the limiter reduce with reduced time step of the model, or

$$\frac{\partial \mathcal{L}}{\partial(\Delta t)} < 1. \quad (6)$$

A weaker than linear dependence of \mathcal{L} on Δt thus retains full convergence, albeit with slower speed than for a limiter that is independent of Δt .

Extensive experience with WAM cycle 4 has shown that this model behaves poorly at short fetches (e.g., Hersbach 1996). This behavior can be attributed to improper scaling behavior of limiter (4). To correct this, several alternative limiters have been proposed for WAM cycle 4. These limiters generally replace the Phillips type spectral shape of limiters (2) and (4) with a Toba type spectral shape, and replace $\Delta t/\tau$ with a properly scaling linear dependence on Δt (e.g., HJ99, Luo and Sclavo, 1997, Hargreaves and Annan 1998, Monbaliu *et al.*, 2000). A good example of such a limiter is the HJ99 limiter, which is given as

$$\mathcal{L}_4 = 3.0 \times 10^{-7} g u_* f^{-4} f_c \Delta t, \quad (7)$$

where u_* is the friction velocity, limited by its Pierson Moskowitz value, and f_c is the dynamically calculated cut-off frequency as described above. The test cases presented in the above papers suggest that such limiters indeed result in a small dependence on the numerical time step Δt , and show proper scaling behavior for short fetches. HJ99 furthermore justify the use of a nonconvergent limiter by stressing the importance of proper scaling behavior of the model while adhering to large time steps, and by mentioning our general lack of understanding of the physics in the spectral range where the limiter is usually activated. It therefore seems that at least for engineering purposes, where the bottom line is that the model performs properly, reliably and economically, the most recent limiters like (7) are a major step forward for WAM, and potentially for other third-generation wave models.

From a scientific point of view, however, the nonconvergence of all new limiters violates rudimentary principles of numerical modeling. It is also contrary to the philosophy of third-generation wave models. This philosophy implies that the evolution of the spectrum should be determined by the parameterization of the source terms alone. This philosophy was adopted in order to get the best possible wave model, that could also be used as a tool to investigate alternative formulations for source terms. If the limiter becomes a systematic part of the solution, it becomes difficult if