The Global Atmosphere and Ocean System, 1998, Vol. 6, pp. 295–309
Published by license under
the Gordon and Breach Science
Publishers imprint.
Printed in India.

MODELING WIND WAVES USING WAVENUMBER-DIRECTION SPECTRA AND A VARIABLE WAVENUMBER GRID*

HENDRIK L. TOLMAN[a,†] and NICO BOOIJ[b]

[a] *UCAR visiting scientist, Ocean Modeling Branch, Environmental Modeling Center, NOAA/NCEP;*
[b] *Section Fluid Mechanics, Department of Civil Engineering, Delft University of Technology*

(Received 10 August 1997; In final form 15 August 1998)

The use of the wavenumber-direction spectrum in wind wave models results in an effective loss of model resolution for waves traveling from deep to shallow water and in additional numerical disadvantages, when a conventional invariant spectral grid is used. In this paper we present a theoretical study of how the effects of variable depths and currents may be incorporated in a spatially varying wavenumber grid. It is shown that effects of currents cannot be efficiently incorporated in the grid. Effects of variable depths are incorporated in a wavenumber grid which is equivalent to a spatially invariant frequency grid. The resulting equations are nearly identical to the conventional equations for the frequency-direction spectrum, but include a more elegant way to address effects of temporal variations of the water depth. Furthermore, the technique employed to derive the equations for the variable grid approach closely resembles that used in the conversion between different spectral descriptions. The formulations presented in this paper may therefore serve as a basis for discussion of the selection of spectral descriptions.

Keywords: Wind waves; wind wave modeling

1. INTRODUCTION

Wind waves in shelf seas and oceans are historically described using their surface elevation spectrum as a function of frequency f, as can be obtained from a time series analysis of the water level elevation at a single point. A

*OMB contribution Nr. 162.
†Corresponding author. 5200 Auth Road Room 209, Camp Springs, MD 20746.

similar approach was adopted in most early spectral wave models, which usually solve an energy balance equation for the two-dimensional spectrum $F(f, \theta)$, where θ represents the direction of the spectral component (see, for instance, SWAMP Group 1985). From a theoretical point of view, however, the wavenumber spectrum $F(\mathbf{k})$ or $F(k, \theta)$ has been considered more appropriate for modeling wind waves due to its invariance properties with respect to the water depth for the physics of wave growth and decay (*e.g.*, Kitaigorodskii, 1962, 1983; Kitaigorodskii *et al.*, 1975; Bouws *et al.*, 1985). For this reason, the wavenumber-direction spectrum has been used to describe the wave field in several recent wave models (*e.g.*, Abdalla and Ozhan, 1993; Jansen *et al.*, 1993; Van Vledder and Dee, 1994). For such models the wavenumber-direction spectrum $F(k, \theta)$ is generally preferable to the vector wavenumber spectrum $F(\mathbf{k})$, because discretization of the latter spectrum leads to a directional anisotropy of the spectral resolution.

When a component of a wave field travels from deep to shallow water ('shoaling'), its wavenumber undergoes large kinematic variations. This is illustrated in Figure 1, which shows characteristics in wavenumber-depth $(k-d)$ space for waves propagating in water with varying but steady depths without mean currents (solid lines). In such conditions, the frequency f remains invariant, and k follows directly from d and the dispersion relation

$$\sigma^2 = gk \tanh kd, \tag{1}$$

where a $\sigma = 2\pi f$ is the radian (intrinsic) frequency. The characteristics in Figure 1 correspond to a set of discrete frequencies, representative for the spectral discretization in ocean wave models. The discrete wavenumber grid of a numerical wave model is usually kept constant throughout the model (illustrated by the dotted lines in Fig. 1). Shoaling then leads to two numerical disadvantages.

First, the kinematic wavenumber variations compress the wave field in k-space for decreasing depths, resulting in an effective loss of resolution in shallow water. This is quantified in Table I, which presents the resolution of an invariant k-grid relative to that of an invariant σ-grid for several wave periods and water depths, assuming identical resolutions in deep water (upper left corner of table). For intermediate depths, the resolution of the wavenumber grid actually improves somewhat (up to 20% for $kd \approx 1.2$). For shallow water, however, (lower right side of table), the relative resolution of the wavenumber grid deteriorates dramatically (by more than 50% for $kd < 0.27$). Particularly swell energy is generally poorly resolved by commonly used spectral resolutions. To assure that the spectral

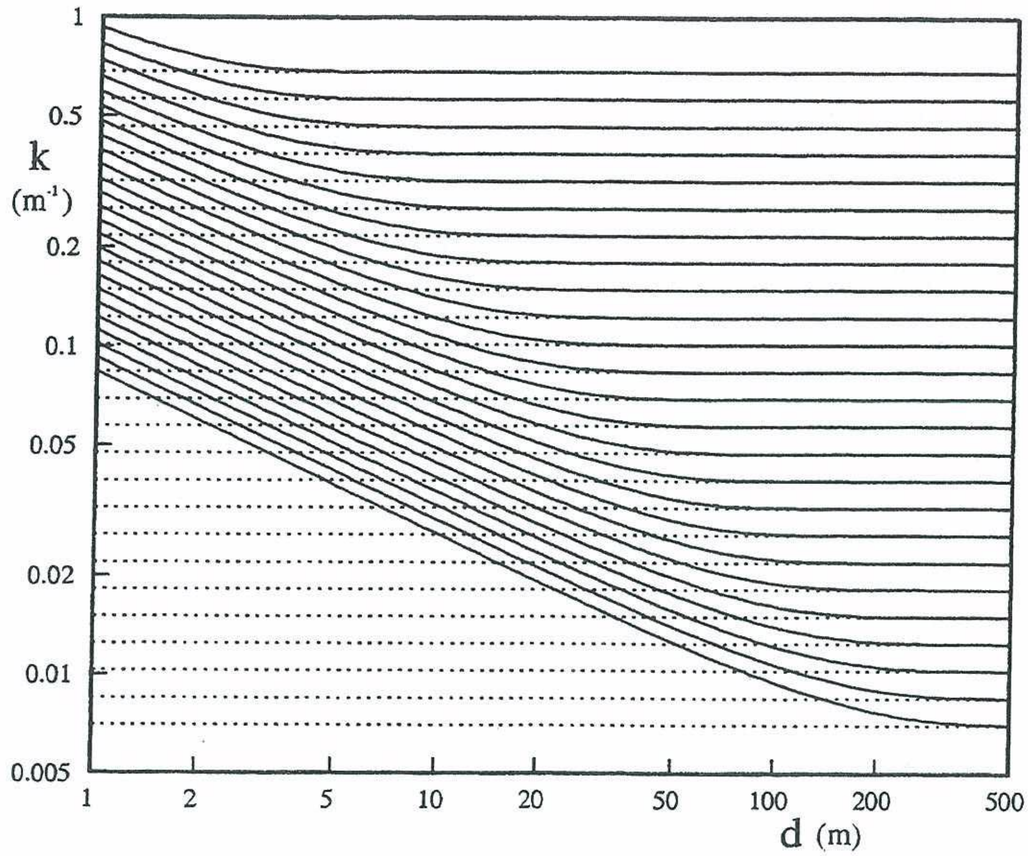

FIGURE 1 Characteristics (solid lines) in wavenumber-depth ($k - d$) space for quasi-steady water depth variations in conditions without mean currents. The dotted lines represent an exponential wavenumber grid corresponding to a frequency grid as representative for numerical wave models ($f_{i+1} = 1.1\, f_i$, with f ranging from 0.041 Hz to 0.41 Hz).

TABLE I Resolution of an invariant wavenumber grid relative to an invariant relative frequency grid for several wave periods T and depths d. Relative resolution defined as inverse ratio of grid increments Δk or $\Delta\sigma$. Identical deep-water resolutions

T(s)	*d(m)*					
	80	40	20	10	5	2.5
5	1.00	1.00	1.02	1.15	1.19	1.03
10	1.02	1.15	1.19	1.03	0.81	0.60
20	1.19	1.03	0.81	0.60	0.44	0.31

resolution for target swell periods of the k-grid is comparable to that of a σ-grid, the number of discrete wavenumbers has to be increased by the inverse of the normalized resolutions presented in Table I. This will obviously have a significant impact on memory and run-time constraints of a practical model.