Comments on "The Goddard Coastal Wave Model. Part II: Kinematics" *

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1. Introduction

Lin and Huang (1996b, hereafter denoted as LH) discuss kinematic aspects of the new Goddard Coastal Wave Model (GCWAM). Their model is compared to the well-known WAM model (WAMDI Group 1988; Komen et al. 1994) and to the WAVEWATCH model (Tolman 1991b, 1992) using theoretical considerations and test cases. The basic equation of GCWAM is given as [LH, Eq. (11)][1]

$$\frac{\partial A}{\partial t} + \frac{\partial c_{g\lambda} A}{\partial \lambda} + \frac{1}{\cos\phi}\frac{\partial c_{g\phi} A \cos\phi}{\partial \phi} + \frac{\partial c_{\theta} A}{\partial \theta} + \frac{\partial c_{\omega} A}{\partial \omega} = S, \tag{1}$$

which is identical to the governing equation of WAVEWATCH[2] and represents an extension of the governing equation of WAM. The left side of this equation represents the effects of wave propagation as dictated by the dispersion relation and is inherently linear. The right side represents source and sink functions, including several effects of nonlinear wave propagation. Focusing on kinematic aspects of wave propagation, LH assume $S \equiv 0$ throughout the paper.

In WAM and WAVEWATCH purely linear propagation is considered, and the characteristic velocities c_g, etc. are based on linear wave kinematics. Lin and Huang (1996b) aim to improve upon this approach by considering nonlinear kinematics in the derivation of the characteristic velocities. As illustrated by Willebrand (1975), such adaptations to Eq. (1) indeed might be important for shallow water, although they are generally irrelevant for deep water. We therefore applaud the attempt by LH. Unfortunately, we disagree with LH on three major points. First, the nonlinear group velocity [LH Eq. (6)] is based on a monochromatic instead of spectral wave concept, and therefore is critically dependent on subjective assumptions. Second, the characteristic velocities c_θ and c_ω [LH Eqs. (17) and (18)] include new terms that are attributed to nonlinear kinematics, but that appear to be inconsistent with conventional nonlinear expansion and with elementary physical properties of waves. Third, previous wave models are seriously misrepresented, and appropriate comparisons with previous models are not presented in the test cases. We will discuss these three points in the following sections and provide conclusions.

2. Group velocity

In their Eq. (6), LH present their nonlinear group velocity c_g. This equation is based on a monochromatic nonlinear dispersion relation [LH Eq. (4), taken from Whitham (1974)]. Application to inherently random wind waves requires subjective assumptions (as will be discussed below). It appears more appropriate to start with a spectral model for the wave field and allow for nonlinear interactions between all spectral components. Willebrand (1975) shows that such an approach not only leads to conventional resonant interactions[3] but also

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[1] The notation of LH is adopted throughout this discussion.

[2] The equation for WAVEWATCH is originally expressed for a plane grid, but this longitude–latitude version is available in the model.

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[3] That is, the nonlinear source terms that are outside the scope of LH.

gives rise to nonlinear corrections to the characteristic velocities in (1). We contend that the latter approach is superior to the former and that it therefore should have formed the basis of GCWAM. Because LH have elected otherwise, their approach requires further inspection.

The nonlinear shallow-water dispersion relation in LH Eq. (4) is a nonlinear series expansion in terms of a small parameter[4]

$$\epsilon = (ka)^2 \tag{2}$$

representing the wave steepness. When applied to a spectral description of the wave field, ϵ has to be replaced by an estimate of the steepness from the spectrum, typically

$$\epsilon = \int_{\delta\theta}\int_{\delta f} k^2 N(f, \theta)\, df\, d\theta, \tag{3}$$

where k^2N is the steepness spectrum, and where $\delta\theta$ and δf are integration bands for which estimates have to be provided.[5] Consistent with the spectral description of wind waves, an infinite number of wave components can be considered with $\delta\theta, \delta f \rightarrow 0$. Consequently, $\epsilon \rightarrow 0$, and this approach reproduces linear propagation (consistent with the inherently linear nature of the spectral description of a random wave field). Finite nonlinear expansion terms can only be obtained if finite bandwidths $\delta\theta$ and δf are retained. The magnitude of the nonlinear corrections then strongly depends on the arbitrary choice of the bandwidth, or following LH, on the arbitrary spectral resolution.[5] This is obviously aphysical, and LH Eq. (4) should therefore not be used as the basis of a nonlinear expansion for the group velocity for irregular waves and cannot be considered an improvement over the well-established linear approach as used in previous models.

3. Refraction and frequency shift

Equations (17) and (18) present the characteristic propagation velocities c_θ and c_ω of GCWAM. These equations are given as

$$c_\theta = \cdots + \frac{1}{k}(c_g - c)\frac{\partial k}{\partial n}, \tag{4}$$

$$c_\omega = \cdots + (\mathbf{c}_g + \mathbf{V})\cdot\nabla(\sigma + \mathbf{k}\cdot\mathbf{V}), \tag{5}$$

where $\cdots$ represent the conventional linear terms as presented by, for instance, LeBlond and Mysak (1978, §6), Christoffersen (1982), Mei (1983, p. 96), or Tolman (1990b),[6] and where the additional terms are the ''new'' terms of LH.

Equation (5) or LH Eq. (18) is correct, provided that the spatial and temporal dependence of $\mathbf{k}$ [and a, if such a dependence is assumed as in LH Eq. (9)] is treated explicitly. If the new term in Eq. (5) is expanded allowing for these dependencies, it vanishes with the use of the proper form[7,8] of LH Eq. (10). Alternatively, x and k may be considered as independent variables in the derivation of the characteristic velocities, in which case Eq. (5) can be written as

$$c_\omega = \frac{D\omega}{Dt} = \frac{\partial\omega}{\partial t} + \dot{\mathbf{x}}\cdot\nabla_x\omega + \dot{\mathbf{k}}\cdot\nabla_k\omega. \tag{6}$$

The first and second term on the right are identical to $\cdots$ and the ''new'' term in (5). Using the equations for the conservation of waves and the irrotationality of the wavenumber vector,[8]

$$\dot{\mathbf{k}} = \frac{D\mathbf{k}}{Dt} = -\nabla_x\omega. \tag{7}$$

Furthermore, $\dot{\mathbf{x}} = \nabla_k\omega$, so that the second and third terms in Eq. (6) cancel. Thus, for steady media, $c_\omega = 0$, contrary to the assertion in LH. This will be true even if it is assumed that ω and c_ω explicitly depend upon the wave amplitude (i.e., in a nonlinear approach).

We have not been able to trace the origin of the new term in Eq. (4). If, however, x and k again are taken as independent variables, $\partial k/\partial n \equiv 0$, and the new term vanishes. Thus, the new terms in Eqs. (4) and (5) appear to be due to mathematical errors.

Lin and Huang appear to justify the new terms by labeling them as effects of nonlinear kinematics.[9] The magnitude of terms introduced by a nonlinear series expansion are expected to depend on the wave energy or steepness and vanish for $N \rightarrow 0$. The new term in Eq. (4), however, is always finite for shallow water with varying depths, as k, $(c_g - c)$ and $\partial k/\partial n$ then are all finite. On p. 856 LH furthermore point out that the new term in Eq. (5) is similarly finite in shallow water for the linear test case 1. It therefore appears inconsistent to claim that both terms are expressions of nonlinear wave kinematics.

It should finally be noticed that the new frequency shift term results in aphysical model behavior. Consider, for instance, an old swell field approaching the coast in

[4] Formally, $\epsilon = ka$, (2) is adopted for convenience of notation.

[5] LH Eq. (6) is dimensionally inconsistent because the integration over the spectrum has been omitted. In replies to an earlier version of this comment, Lin identified this as a typographical error and stated that the spectral density N should be replaced by $\hat{N} = \int_{\delta\theta}\int_{\delta f} N(f, \theta)\, df\, d\theta$ throughout LH Eq. (6). The integration increments are defined by the spectral resolution and are given as $\delta\theta = 15°$ and $\delta f = 0.1f$.

[6] Assuming that n is a coordinate perpendicular instead of tangential to $\mathbf{k}$.

[7] LH mistakenly use the intrinsic frequency σ rather than the apparent frequency ω in their Eq. (10). The proper form of this equation follows from the conservation of waves.

[8] The conservation of waves and the irrotationality of $\mathbf{k}$ follow directly from the definitions of $\mathbf{k}$ and ω in the phase function of monochromatic waves or spectral components, for example, Phillips (1977), LeBlond and Mysak (1978), and Mei (1983).

[9] Whereas this was not clear to us from LH, this argument is repeatedly used in the reply to the original version of this discussion.

intermediate or shallow water. Such swell fields are essentially monochromatic, and their low steepness implies linear dispersion. According to LH Eq. (18) and the corresponding discussions on pp. 852 and 856, the absolute frequency ω of this swell field will decrease while approaching the coast. This implies that some individual waves disappear, which is obviously aphysical for near-monochromatic waves.

4. Previous models and test cases

Lin and Huang (1996b) refer to two previous third-generation wave models: WAM (WAMDI Group 1988; Komen et al. 1994) and WAVEWATCH (Tolman 1991b, 1992).[10] Both models, however, are seriously misrepresented.

The governing equation for WAM in shallow water applications is given as [WAMDI Group 1988, Eq. (2.1); Komen et al. 1994, Eq. (3.33) and section III.4][11]

$$\frac{\partial N}{\partial t} + \frac{\partial c_{g\lambda} N}{\partial \lambda} + \frac{1}{\cos\phi}\frac{\partial c_{g\phi} N \cos\phi}{\partial \phi} + \frac{\partial c_\theta N}{\partial \theta} = 0, \qquad (8)$$

where N is the energy density spectrum. In form, this equation is similar to the balance equation of GCWAM [LH Eq. (11)]. Erroneously, LH represent their Eq. (12) as the governing equation of WAM. In the latter equation the characteristic velocities are placed outside the partial derivatives (transport equation form). That WAM uses a form where characteristic velocities are included inside the derivatives (conservation equation form) is clear from the numerical implementation of Eq. (8) (Komen et al. 1994, p. 237; or source code of WAM).

Lin and Huang furthermore suggest that cycle 4 of WAM has been used in their test cases,[12] but state that their version of WAM ''. . . does not include wave-current interaction or depth change . . .'' (p. 856). This implies that they have applied the deep water version of WAM to all their shallow water test cases. To our knowledge, all released versions of cycle 4 of WAM include shoaling and refraction as part of the linear kinematics. It is up to the user to select either the deep water or the shallow water version. To suggest that the results of LH Fig. 2 are representative for WAM cycle 4 is therefore misleading. Figure 3 of LH appears to be dominated by the shift of energy to directions normal to the shoreline, corresponding to Snel's law. When implemented properly, WAM cycle 4 also describes this effect (in spite of the diffusive numerics), as is illustrated in Fig. 4.48 of Komen et al. (1994, p. 346).[13]

Finally, modifications to WAM cycle 4 to include current refraction are available from MPI Hamburg. Due to the present structure of the source code, WAM can only deal with steady currents. This implies that WAM can also include the steady currents of test 2, and at least qualitatively represent the corresponding current refraction, but is (presently) not able to deal with current refraction due the the unsteady currents in test 3.

The WAVEWATCH model (Tolman 1991b, 1992) is only referred to in passing, but is generally ignored. This is justified in Lin and Huang (1996a) and in a reply to a previous version of this discussion by claiming that this model is unconditionally unstable. This seriously misrepresents WAVEWATCH as is discussed in Tolman et al. (1998).

WAVEWATCH uses the same conservation equation as GCWAM [Eq. (1)], basing $c_{g\lambda}$, $c_{g\phi}$, c_θ, and c_ω on conventional linear kinematics. Test 1 of LH is dominated by linear refraction and should therefore reproduce Snel's law and linear shoaling. Even in its original version with somewhat diffusive numerics, WAVEWATCH describes Snel's law excellently (Tolman 1991b, Fig. 5).[13] Lin and Huang's test 2 adds effects of a steady current. Applications of WAVEWATCH to steady cases can be found in Fig. 7 of Tolman (1991b) or in Holthuijsen and Tolman (1991). Finally, test 3 of LH considers unsteady currents. WAVEWATCH was designed particularly to investigate effects of unsteady tidal currents on wind waves (Tolman 1990a, 1991a). Considering these examples, it would have been appropriate to compare GCWAM directly to WAVEWATCH.

5. Discussion and conclusions

Considering the above, GCWAM can be considered as an attempt to expand upon the WAVEWATCH model by including nonlinear kinematics. Because these extensions appear to be flawed, GCWAM cannot be considered state-of-the-art or an improvement compared to WAVEWATCH. The development of GCWAM nevertheless raises the question whether it is necessary to include nonlinear kinematics in coastal wave models. Whereas nonlinear kinematics are likely important in studying higher-order nonlinear wave–wave interactions, nonlinear extensions to the characteristic propagation velocities are not necessarily important for a general purpose wave model. Such a model should at least describe all relevant processes at the lowest order possible. This implies that a coastal wave model should at least include a linear description of wave propagation over inhomogeneous and unsteady depths and currents

[10] Not described as such, but references to papers included.

[11] Because WAM considers steady media only, $c_\omega \equiv 0$, and the corresponding term in this equation can be omitted; source terms omitted as in LH.

[12] See labels in LH Fig. 2.

[13] Contrary to claims by Lin, refraction test results of WAM and WAVEWATCH have been obtained with $S \equiv 0$ in the corresponding balance equations.

(including shoaling, refraction, and the momentum exchange between waves and currents), quasi-linear wave growth and dissipation, and higher-order wave–wave interactions to represent the lowest-order mechanism to shift energy to low frequencies during wave growth. Until all these mechanisms are adequately understood at their lowest-order approximation, higher-order approximations for parts of the model are not likely to significantly improve general model behavior. Presently, the weakest link in wave models is the dissipation or ''whitecapping'' term, which is generally used as the closure term in tuning wave growth behavior. Until this term is understood and can be modeled on more physical grounds, it is unlikely that improved kinematics will significantly improve overall model behavior. In fact, only modest progress has been made in this aspect of wave modeling since Willebrand (1975) closed his paper with a similar statement.

6. Postscript

From this discussion and the accompanying reply by Dr. Lin it is obvious that we disagree on many points. In our opinion, this disagreement is largely based on erroneous claims by Dr. Lin. For many claims, we have not been able to find support in the references provided by Dr. Lin. Furthermore, several statements appear to be erroneously attributed to us, and we have not been able to reproduce most of the results that Dr. Lin attributes to ''our'' models and schemes. We have chosen not to discuss this in our comments, as it would seriously distract from our reservations with the original papers. Considering the above, we urge the reader not to take any statements in the comments and replies at face value, but to independently check all arguments, references, and results.

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