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TECHNICAL NOTE ¹

**PARAMETERIZATION OF MIXING IN UPPER
OCEAN**

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- No. 2. Richardson, W. S., D. J. Schwab, Y. Y. Chao, and D. M. Wright, 1986: Lake Erie Wave Height Forecasts Generated by Empirical and Dynamical Methods -- Comparison and Verification. Technical Note, 23pp.
- No. 3. Auer, S. J., 1986: Determination of Errors in LFM Forecasts Surface Lows Over the Northwest Atlantic Ocean. Technical Note/NMC Office Note No. 313, 17pp.
- No. 4. Rao, D. B., S. D. Steenrod, and B. V. Sanchez, 1987: A Method of Calculating the Total Flow from A Given Sea Surface Topography. NASA Technical Memorandum 87799, 19pp.
- No. 5. Feit, D. M., 1986: Compendium of Marine Meteorological and Oceanographic Products of the Ocean Products Center. NOAA Technical Memorandum NWS NMC 68, 93pp.
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Parameterization of mixing in upper ocean²

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Abstract

Because boundary layers with small thermal and mechanical inertias approximate steady-state conditions, the associated density and momentum fluxes tend to be constant with depth. As a result, these fluxes may be chosen as external parameters, and it then becomes possible to apply Monin-Obukhov similarity theory. For fluids with large thermal inertias such as the ocean, the density flux is a function of depth; thus, the external thermal forcing is no longer a governing parameter. In addition, if the mechanical inertia is also large, the structure of the boundary layer is not universal because it depends on the previous evolution of the thermal and mechanical forcing. However, if the mechanical inertia is small, the dynamical structure of the boundary layer adjusts almost instantaneously to the density structure and the mechanical forcing. This property allows us to generalize the Monin-Obukhov theory for stratified boundary layers through specification of a stratification parameter which characterizes the internal density structure instead of the external density flux. The thermal time scale for the upper ocean is relatively large whereas the dynamical time scale is 1-2 orders of magnitude shorter. Consequently, the upper ocean may be considered steady-state in a dynamical sense and any dynamical property depends primarily on the depth, surface momentum flux, and the vertical density structure. Based on this assumption, we can choose the appropriate scaling for the turbulent mixing coefficient. A nondimensional turbulent mixing coefficient is derived which depends on the stratification parameter which, in turn, includes the surface stress and the integral density deficit for the entire layer above. The

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general form and asymptotic behavior of the nondimensional turbulent mixing coefficient as a function of the stratification parameter is formally obtained using dimensional considerations. Its structure is determined using empirical data obtained in the atmospheric boundary layer. The final formulation is based on 8 years of temperature profiles acquired at Ocean Weather Ship (OWS) PAPA. This approach reproduces the 8-year evolution of the upper ocean with a maximum *rms* difference of approximately 1 K and a bias of 1 K over the depth range of 0-150m. An additional 1-year simulation of the upper ocean at OWS CHARLEY and a 9-year simulation at OWS NOVEMBER suggest the universal nature of this approach. Overall, the simple turbulent mixing scheme which is derived reproduces the evolution of the upper ocean with accuracies similar to those obtained using more complicated models. Related problems of sea surface temperature data assimilation and its predictability are also considered. In addition, an investigation of mixed layer deepening forced by constant wind and mass exchange is performed. Finally, estimates of the influence of vertical resolution on the accuracy of model simulations are obtained.

1. Introduction

The most important ocean characteristics which is responsible for ocean-atmosphere thermodynamic interaction is sea surface temperature (SST). The primary spatial and temporal variations in temperature are concentrated in a seasonally-active upper layer, whose depth is mainly determined by the amplitude of the seasonal variations in mass exchange, intensity of wind mixing, and vertical motion. This active layer is usually topped by a so-called mixed layer (ML), where vertical gradients of temperature are very small. The physics of ML are similar to that of atmospheric surface layer but with specific features which are different due to high thermal inertia of water (see discussion in section 2). Assuming that vertical fluxes of heat and momentum in the atmospheric surface layer are of the same order, we can estimate that ratio r of vertical temperature differences in water ΔT_w and in air ΔT_a

$$r = \frac{\Delta T_w}{\Delta T_a} = \frac{c_a}{c_w} \sqrt{\frac{\rho_a}{\rho_w}} \quad (1)$$

(c_w , ρ_w and c_a , ρ_a are specific heat capacity and density of water and air correspondingly) is of order 0.01. This does not mean, however, that stratification the ML is neutral, because the ratio of Monin-Obukhov scales for water L_w and for air L_a are

$$\frac{L_w}{L_a} = (\alpha T)^{-1} \frac{c_w}{c_a} \sqrt{\frac{\rho_a}{\rho_w}} \quad (2)$$

($\alpha \simeq 2 \cdot 10^{-4} C^{-1}$ is a coefficient of thermal expansion of water) and turns out to be of order 1, and thus the role of stratification in the ML is comparable to that in the atmospheric surface layer. Consequently, the ML may be considered homogeneous only in a sense that the gradients of temperature here are much less than it is below the ML in the upper thermocline, where the temperature gradient is large (approaching values of $1 K m^{-1}$) and the Richardson number (Ri) is usually high. Nevertheless, even under very stable conditions, the heat and mass fluxes still greatly exceed those which result from molecular diffusion, and significant variations in temperature in upper thermocline are usually observed. Therefore, without considering heat transfer in upper themocline, it is not possible to reproduce the variations

of SST. Although the mechanism of heat exchange between ML and upper thermocline is understood poorly, several processes are most likely responsible for producing vertical heat exchange between ML and upper thermocline (Large et al, 1994, Garret, 1995):

1. After the decrease in dynamical forcing or an increase in heating, new discontinuities in density form which result in the trapping of lower part of ML and its assimilation into upper thermocline.
2. Pure turbulent entrainment is produced by intense mechanically-generated or convective turbulent eddies due to strong winds and/or surface cooling.
3. Mixing is produced by breaking of internal waves. Measurements show that this type of mixing is always present in ocean interior (Gregg, 1987). The ensemble effect of this intermittent turbulence produces a momentum and heat exchange between ML and upper thermocline and vertical fluxes in ocean interior.
4. Mixing is produced by surface waves. Nonlinear surface waves can produce mixing and also internal waves which finally break, creating turbulence.
5. Mixing is produced by irreversible components of secondary motions such as Langmuir circulation, resulting from dynamic instability of ML which may be coupled to the surface waves.

Because SST is very sensitive to vertical mixing in upper layer, ocean models should clearly be able to parametrize this process. Generally, the term '*parameterization*' may be defined as a formulation of rules for modeling the subgrid processes responsible for energy transformations in terms of the large-scale variables. For ocean models, 'large-scale' corresponds to variables which are averaged over 4-D space-time boxes with horizontal size of order of 1-100 km, vertical size of order of 1-10 m and time intervals of order 1 min-1 hour. The specific feature of upper ocean is its well pronounced fine structure and multiscale space and temporal variability (Monin et al. 1977, Fedorov XXX),. Horizontally averaged fields loose this structure and thus oversimplify the real processes. This dramatic gap between ocean model scales and the actual scales observed 'in situ' reduces the parameterization

problem to purely phenomenological, consisting of the construction the empirical relationships between averaged external forcing and related large-scale variations of the upper ocean. Thus, observational data which are needed to formulate and validate various parameterizations should encompass large time and space scales. Space-averaged data (except for remotely sensed SSTs) are usually unavailable, and so, long series of temperature, salinity (and current) profiles should be used. Such data have only been obtained at Ocean Weather Stations (OWS), where where long-time series of temperature profiles were obtained. Most models of of the ML have been developed using these data in spite of their relatively low accuracy.

A simulation of upper ocean is based on heat conductivity equation. ⁵

$$\frac{\partial T}{\partial t} = -\frac{\partial \overline{w'T'}}{\partial z}, \quad (3)$$

where u, u', w', T' have standard definitions, and the axis z points up.

Eq. (3) assumes horizontal homogeneity. In general case, this equation should incorporate the effects of 3-D advection and horizontal mixing, thus, Eq. (3) may be also treated as a 'submodel' of a general circulation ocean model which uses numerical splitting method.

Models of upper ocean may be divided into two groups: (1) *parametric* and (2) *differential*. In *parametric* (or so-called 'bulk') models, the general structure of upper ocean is assumed to be known, and as a result equations can be derived for the parameters describing this structure. The *parametric* models take their origin from Kraus and Turner (1967) which first introduced the concept of a mixed layer characterized by temperature and depth, and then derived the equations for describing its evolution. Most parametric models contain only the following parameters: depth of mixed layer h , its temperature T_s , and certain characteristics of upper thermocline which are assumed to be known: temperature difference at the bottom of mixed layer $T_s - T_h$, or vertical temperature gradient in upper thermocline $\left(\frac{dT}{dz}\right)_{h+0}$, or

⁵For simplicity, we consider in this introduction only thermal processes but generally upper ocean models should take into account salinity as well. Volume heating by short-wave radiation is also neglected

both these parameters. One evolutionary equation may be obtained by integrating (3) over depth up to $z = h$ taking into account the variability of h .

$$h \frac{\partial T_s}{\partial t} = -(T_s - T_h) \frac{dh}{dt} - \overline{w'T'}|_{z=0} + \overline{w'T'}|_{z=h} \quad (4)$$

Here T_h is the temperature at the bottom of ML. First term in (4) describes the heat exchange between ML and underlying thermocline produced by displacement of lower boundary of ML. This term can be interpreted only when $\partial h/\partial t > 0$. In the opposite situation when $\partial h/\partial t < 0$, a formal use the (4) would assume that process inverse to mixing occurs, when cooled by $T_s - T_h$ water is transferred to the underlying region. Instead, Kraus and Terrner (1967) suggested that entrainment in this case is absent. In reality, the depth of ML may decrease through the formation of a new temperature discontinuity in the ML arising when the wind weakens or heating increases. Often, a new temperature discontinuity moves down and reaches a base of the ML. The term $-\overline{w'T'}|_{z=0}$ represents the kinematic heat exchange through the surface ⁶, and $-\overline{w'T'}|_{z=h}$ describes heat diffusion through the bottom of ML at $z = -h$. Since this term is unknown, further assumptions are related to its parameterization.

A simple way to derive an additional equation consists of multiplying equation(3) by z and again integrating over depth. As a result, we obtain an equation for a quantity which is proportional to potential energy for a unit column of depth h :

$$\frac{h}{2} \frac{\partial T_s}{\partial t} + (T_s - T_h) \frac{\partial h}{\partial t} = \overline{w'T'}|_{z=h} - \int_{-h}^0 \overline{w'T'} dz \quad (5)$$

This equation again contains unknown variables $T_s - T_h$, $\overline{w'T'}|_{z=h}$ and the term $\int_{-h}^0 \overline{w'T'} dz$ which is proportional to a mutual transformation of potential energy and kinetic energy of turbulence. This term may be obtained from integrating the equation of kinetic energy of turbulence over the depth of ML. This equation introduces dynamic forcing through the action of wind stress and turbulent energy flux from dissipating waves. Unfortunately, the

⁶Strickly speaking, due to surface kinematic boundary condition, this term becomes zero at interface. Thus, heat exchange through the surface may occur by molecular conductivity (and also from incoming solar radiation and by sprays)

final equations cannot be derived formally because equation for turbulent energy contains nonlinear production and dissipation terms and last of them has singularity near the surface. Thus, the relations so obtained are qualitative, leaving ample room for different 'ad hoc' constructions. As a result, a large number of schemes for parameterizing the oceanic and atmospheric mixed layers have appeared.

Kitaigorodskii and Miropolskii (1970) used a different approach for parameterizing the structure of seasonally active layer based on similarity considerations. They assumed that continuous temperature profiles in seasonal active layer may be described as

$$T = \begin{cases} T_s, & z \leq h_m \\ T_s - (T_s - T_a)M(\zeta), & h_m < h < h_a \end{cases} \quad (6)$$

where unknown variables are still ML depth, h_m , and the ML temperature, T_s . The external parameters are depth of the seasonally active layer h_a and the temperature at the bottom T_a of this layer, which are assumed to be known or can be derived from a large-scale ocean model. $M(\zeta)$ in (6) is an empirical nondimensional function of nondimensional depth $\zeta = (z - h)/(h_a - h_m)$, approximated as

$$M(\zeta) = \frac{8}{3}\zeta - 2\zeta^2 + \frac{1}{3}\zeta^4 \quad (7)$$

Relations (6), (7) embrace a large variety of temperature profiles in upper ocean. Advantage of this approach is that: (1) due to discontinuity of temperature profile (6) within the active layer, no hypothesis for turbulent entrainment is required, and (2) variability of T_a is much less than that of T_h in previous scheme. Of course, uncertainties arising from the integration of the turbulent energy equation over depth arise here also.

Because all parametric schemes use many arbitrary assumptions, this approach may be considered as a pure heuristic, based mostly on empirical data. Numerous calculations (see review by Large et al. 1994) showed that after appropriate tuning ⁷ bulk models (based sometimes on different assumptions)

⁷A delicate process consisting of modifying the formulation of a model to obtain better results and also indirectly to take into account effects which were not included. Models which need tuning for simulating different data sets cannot be used for parameterizing.

are able to reproduce the evolution of temperature and depth of mixed layer satisfactorily. An advantage of this approach is its computational efficiency. The mathematical formulation of these models is simple, and it may be fully analysed for obtaining fast computing and robust numerical scheme. Nevertheless, there are disadvantages of this approach also:

1. Oversimplification of the structure of upper ocean. In reality, temperature profiles are continuous, so, the definition of h_m and $T_s - T_h$ (or T_a and h_a) cannot be made objectively.
2. Oversimplifying the heat exchange mechanisms between ML and upper thermocline, and neglecting thermal conductivity in the upper thermocline.
3. Inconsistencies arise with differential formulations of ocean models. If gridded vertical temperature profiles are treated as step functions, it is unclear which temperature discontinuity corresponds to the lower boundary of ML. Truncated temperature profiles with the same accuracy may be treated also as continuous functions, thus, the ML depth cannot also be derived objectively again. As a result, the uncertainties arise in combining differential ocean models with bulk model of upper ocean.

In fact, the bulk models are well suited for describing the evolution of ML per se but they do not fully describe the evolution of upper ocean which results from turbulent exchange between ML and upper thermocline.

The alternative approach, based on differential equations and closure hypotheses, may be applied formally for the evolution of the entire upper ocean. Nevertheless, the applicability of this approach to modeling turbulent entrainment and vertical diffusivity in upper thermocline is very questionable.

It is well known that all models of turbulent flow which are based on the Friedman-Keller equations (Monin and Yaglom, 1971) assume the existence of fully-developed turbulence which produces a cascade of energy from the mean flow to large and small eddies. The presence of fine structure and the almost complete absence of turbulence below ML shows that these fundamental assumptions are violated. Thus, models which are based on closure schemes are realistic only within ML and within turbulent patches

which arise, for example, from breaking the internal waves. It should not be assumed that the equations for diffusion of temperature and momentum together with the equation for turbulent energy, rate of dissipation, length scales relationships, etc, can adequately describe the processes of entrainment and mixing under stable stratification. First, the effects of intermittency of mixing, statistical dynamics of internal waves and coherent structures are completely not represented in any turbulent closure model. Second, all of these systems of equations predescribe the disappearance of turbulence when the rate of the transformation of turbulent kinetic energy to potential energy approaches to the rate of generation. This is true from the point of view of conventional theory of turbulence, but it is not correct physically, because turbulence in this case continues to act, but in a transformed and poorly understood manner (e.g. Gregg, 1987, Muller, 1993). Also, it is unlikely that by adding more complexity to existing closure schemes is possible to reach better results.⁸ If these models are applied to entire upper ocean, they are likely to fail, because they ignore the specific physics of mixing under stable stratification, which, however, is taken into account in the bulk models.

It may be shown that numerical models of ML based on differential approach in some cases do not approximate an initial differential equations. Here we consider only one aspect of the problem - turbulent entrainment at the base of ML. Assume that a mixing model, based on an appropriate closure scheme, is able to describe the general features of turbulence for small positive Ri numbers. As we approach to the upper thermocline, Ri continues to increase, and eventually reaches the critical value Ri_{cr} where the turbulent coefficient approaches zero or is replaced by some small 'residual' value, which is essentially a tuning parameter. During periods of heating when the ML is well pronounced and upper thermocline has a sharp gradient, this transition occurs very quickly, sometimes within a single vertical step in a numerical model. In this case, the evolution of the depth of ML depends completely on the details of the numerical scheme. In schemes where the gradient of temperature and the turbulent coefficient are calculated at

⁸For example, Kantha and Clayson (1995) recently suggested a scheme consisting of 6 highly complicated equations supplemented by about 20 additional differential expressions. The equations contain 10 so-called 'constants'. This 'submodel' is more complicated than complete ocean model, and, due to its high order, requires hundreds of gridpoints. This model cannot be fully investigated and its solution may fall into unwanted regimes.

the same point, the rate of displacement for discontinuities depends only on a residual value for turbulent coefficient, thus, it has been introduced manually. In most models, this effect is hidden because staggered grids are used. For such scheme temperature diffusion near density discontinuities is produced not by the limiting value for K but by half of the sum of this value and the adjacent value, which may be not small. Heat transfer is thus enhanced in the upper thermocline, and its value becomes crucially dependent on vertical resolution. Thus, under the same forcing, the depth of ML can grow quickly for coarse resolutions and does not change for fine resolutions (when residual value of K is equal to zero). Thus, the key mechanism for ML evolution is not adequately reproduced in turbulent closure models. Of course, some artificial modifications may be taken to reduce this effect, but in this case precisely these modifications obtain the main importance, and turbulence model itself works only above the entrainment level in ML where mixing is relatively simple. Of course, differential models also may be tuned, but comparison of such models with bulk ML models does not show that either type has obvious advantages. (Martin, 1985).

Thus, we conclude that in both approaches the exchange of heat and mass between the ML and the seasonal thermocline remains an obscure issue. Obviously, it is impossible at present to derive a physical model which describes from first principles the ensemble effect of all processes which may be responsible for vertical mixing in the upper layer. This is why we propose a simple phenomenological approach consisting of the analysis of resulting effects of different and poorly understood processes, in terms of an effective heat conductivity coefficient. Our approach to quantify mixing in the upper ocean is based on a modification to the similarity theory of Monin and Obukhov. This approach is also somewhat analogous to a nonlocal parameterization of turbulent mixing applied to the atmospheric surface layer by Troen and Mahrt, (1986) and recently applied to the oceanic ML by Large et al. (1994). Taking into account the large thermal inertia of the ocean, we have introduced a new nonlocal stratification parameter which makes possible a realistic and very simple description of mixing in the ML and the upper thermocline.

2. Similarity considerations

Similarity theory (Obukhov, 1946; Monin and Obukhov, 1954) has been used extensively to describe the atmospheric surface layer (ASL). However, the general basis for similarity theory must be revised for application to the ocean. Consider the ASL with height $h \simeq 10$ m whose structure is governed by the wind stress τ and the surface heat flux H multiplied by a buoyancy parameter $\frac{g}{T}$ (g is the acceleration of gravity and T is the absolute temperature in K). The averaged equations for temperature and momentum evolution are

$$\frac{\partial T}{\partial t} = \frac{\partial \overline{w'T'}}{\partial z}, \quad (8)$$

and

$$\frac{\partial u}{\partial t} = \frac{\partial \overline{w'u'}}{\partial z}. \quad (9)$$

where u, u', w', T' have standard definitions. These equations allow us to estimate the dynamical (T_{da}) and thermal (T_{ta}) time scales as

$$T_{da} = \frac{\rho_a U_a h}{\tau}, \quad T_{ta} = \frac{c_a \rho_a \Delta T h}{H}, \quad (10)$$

where U_a is the wind velocity, τ and H are the surface stress and heat flux, respectively, ΔT is the vertical temperature difference across the surface layer ($\simeq 1 - 2$ K), ρ_a and c_a are the density and specific heat capacity of air, respectively, and h is a height scale for the ASL. With appropriate values of $\tau \simeq 0.3$ Nm^{-2} , $H \simeq 10 - 100$ Wm^{-2} and $h \simeq 10$ m, (10) gives values for $T_{da} \simeq T_{ta} = 10^2 - 10^3$ s. These time scales indicate that for periods $\geq T_{da}$ and T_{ta} , the thermal and the dynamical structure of the surface layer may be considered to be steady-state. The relatively small thermal and mechanical inertias of the surface layer produce a rapid response to external forcing which explains why Monin-Obukhov similarity theory was generally successful when applied to the lower atmosphere.

Next, we estimate similar time scales for the upper oceanic boundary layer. If we assume that τ and H are the same for both media, then

$$T_{dw} = \frac{\rho_w U_w h}{\tau}, \quad T_{tw} = \frac{c_w \rho_w \Delta T h}{H}. \quad (11)$$

For a vertical temperature difference $\Delta T \simeq 1 \text{ K}$ and a surface current velocity $U_w \simeq .03U_a$ and a layer depth $h \simeq 10 \text{ m}$, the corresponding time scales for T_{dw} are $\sim 10^4 \text{ s}$ and $\sim 10^5 - 10^6 \text{ s}$ for T_{tw} .

As expected, the time scales for the ocean greatly exceed those for the atmosphere. This difference is due to the greater density and heat capacity of water which acts to decrease the rates of momentum and heat evolution. Thus, the steady-state assumption for thermal structure is formally valid only for layer depths of the order of 1-10cm. However, this scale is meaningless in the presence of waves; thus, Monin-Obukhov similarity theory cannot be applied directly to the upper ocean.

A second property of the oceanic boundary layer, which distinguishes it from the ASL, is the large difference between the dynamical and thermal time scales (1-2 orders). This difference arises because the ratio of heat capacity to density is larger for water than it is for air, and because oceanic motions are smaller than atmospheric motions by almost two orders of magnitude.

These scale differences imply that dynamical adjustment of the upper ocean occurs 1-2 orders of magnitude faster than it does for thermal adjustment. A representative value for the dynamical time scale is of the order of several hours. A typical period for significant changes to occur in the atmosphere varies from one to several days; thus, in a dynamical sense, the upper ocean is approximately steady-state. However, the thermal state of the upper ocean is essentially unsteady. As a result, the heat flux is a function of depth and its surface value is not a governing parameter as was assumed when similarity theory was applied to the atmosphere. On the other hand, the surface stress is an important external parameter due to the relatively short dynamical time scale in the oceanic case.

Consider the evolution of the upper oceanic layer with a characteristic depth $h \ll L$ where L is a horizontal length scale $\cong 10^4 - 10^5 \text{ m}$ with a time scale $> T_{dw}$. We assume that kinetic energy is acquired at these scales only from the surface above. The stratification of the upper layer is described by its vertical density distribution $\rho(z, t)$. We now formulate the similarity hypothesis: for steady-state flow, any dynamical property $D(z, t)$ (e.g., turbulent energy, its rate of dissipation, or the turbulent mixing coefficient) may

be represented in the following form

$$D(z, t) = D_*(t) F_D \left(\frac{g}{\rho_0} \rho(z, t), v_*, z \right) \quad (12)$$

where ρ_0 is the mean density of sea water, D_* is an appropriate scale (having the same dimension as D), $v_* = \sqrt{(\tau/\rho_w)}$ is the water friction velocity, and F_D is a function describing the joint influence of dynamical forcing and stratification. The density profile $\rho(z)$ may be represented by a sufficient number of moments of the form $\int_0^z (\rho(y) - \rho(0)) y^n dy$ ($n = 1, 2, 3, \dots$). As a first approximation we assume that the stratification can be represented with reasonable accuracy by a moment of low order. A suitable nondimensional stratification parameter, S_p , may be expressed, for example with $n = 1$, as

$$S_p = \frac{\frac{g}{\rho_0} \int_0^z (\rho(z') - \rho(0)) z' dz'}{z v_*^2}. \quad (13)$$

This parameter represents (to within an arbitrary numerical factor) the ratio of the potential energy of the layer to its kinetic energy. It is positive for stable stratification and vice versa. Monin-Obukhov similarity theory may now be applied in terms of this integral parameter. We can replace the instantaneous values of heat flux by a single parameter which characterizes the entire layer above. This approach represents a reformulation of Monin-Obukhov theory since any result based on this theory can also be expressed in terms of the integral parameter, S_p . For example, the turbulent mixing coefficient, K_T , may be expressed as

$$\tilde{K}_T = \frac{K_T}{\kappa v_*(z + z_0)} = \left(\zeta \frac{\partial F_T}{\partial \zeta} \right)^{-1} \quad (14)$$

where F_T is a function which approximates a nondimensional temperature profile, $\zeta = \frac{z}{L}$ is a nondimensional height, and L is the Monin-Obukhov length scale given by

$$L = \frac{v_*^3}{\kappa \frac{g}{\rho_0} w' \rho'} \quad (15)$$

(See Monin and Yaglom (1971) for details). When $z = 0$, the turbulent mixing coefficient equals $\kappa v_* z_0$. This formal result is useful for approximating temperature profiles for depths where $z \gg z_0$. For wind profiles above the