, K. & Benney, D. The propagation of nonlinear shear flow with a free surface, *Studies in Applied* 76, 69–92.

erical simulation of the kinetic equations for sur- *Izv. Akad. Nauk SSSR. Fiz. Atmosf. Ok.*, 1990, **26**, lish translation: *Izvestia, Atmospheric and Oceanic* no.2, 118–123)

waves on shear currents: Solution of the boundary- *Fluid Mech.*, 1993, **253**, 565–584.

formation of a narrow angular spectrum of wind- nonlinear interaction between wind and waves, *Izv. Fiz. Atmosf. Ok.*, 1989, **25**, no.4, 411–420 (English *Atmospheric and Oceanic Physics*, 1989, **25**, no.4,

ear and Nonlinear Waves, Wiley, New York, 1974,

bility of periodic waves of finite amplitude on the id, *J. Appl. Math. Techn. Phys.*, 1968, **9**, 190–194.

nilga, A.V. On the dynamics of water wave spectra proximation, *Zh. Eksp. Teor. Fiz.*, 1981, **81**, 1318– ation: *Sov. Phys. JETP*, 1981, **54**, 700–706).

ira, V.I. 1990: On the formation of the directional es. *Zh. Eksp. Teor. Fiz.*, 1990, **98**, no.6(12), 1941– tion: *Sov. Phys. JETP*, 1990, **71**, no.6, 1091–1100).

akov, S.V., Novikov, S.P. & Pitaevsky, L.P. *Soli- thod of inverse scattering*, Nauka, Moscow, 1980, slation: Plenum Press, N.Y., 1984, 288 pp.)

slavsky, M.M. Generation and dissipation ranges on of weak turbulence theory for wind waves, *Izv. er. Fiz. Atmosf. Ok.*, 1983, **18**, no.10, 1066–1076 *Izvestia, Atmospheric and Oceanic Physics*, 1983,

ne long-wave cut-off in the wind driven wave *Nauk SSSR Ser. Fiz. Atmosf. Ok.*, 1989, **25**, sh translation: *Izvestia, Atmospheric and Oceanic* –882).

Chapter 7

Direct modeling of one-dimensional nonlinear potential waves

D. Chalikov[a] & D. Sheinin[b]

[a]*UCAR/Ocean Modeling Branch/NCEP, 5200 Auth Road, Camp Spring, MD 20746, USA*

[b]*Massachussets Institute of Technology, Bldg E40-259, 1 Amherst St., Cambridge, MA 02139, USA*

Abstract

A method for numerical investigation of nonlinear wave dynamics based on direct hydrodynamical modeling of 1-D potential periodic surface waves is presented. By a nonstationary conformal mapping, the principal equations are rewritten in a surface-following coordinate system and reduced to two simple evolutionary equations for the elevation and the velocity potential of the surface. For stationary equations, the proposed approach coincides with the conventional complex variable method. For this case, numerical algorithms for solution of gravity (Stokes) and gravity-capillary wave equations are proposed, and examples of numerical solutions are given. The results imply that gravity-capillary waves do not approach Stokes waves as the capillarity coefficient decreases. Both stationary and nonstationary schemes use Fourier series representation for spatial approximation and the Fourier transform method to calculate nonlinearities. The nonstationary model was validated by simulation of propagating waves with initial conditions obtained as numerical (for gravity and gravity-capillary waves) or analytical (for pure capillary, or Crapper's waves) solutions of the stationary problem. The simulated progressive waves did not change their shape during long-term time integration, which indicates high accuracy of the scheme. Another criterion used for model validation was conservation of integral invariants of simulated multi-mode wave fields. A number of long-term model simulations of gravity, gravity-capillary, and pure capillary waves, with various initial conditions, were performed; for the simulated wave fields, distributions of energy and phase speed over fill spectra were analyzed. It was found that the wavenumber–frequency spectra are

[1]OMB contribution no. 131.

l separated into patterns lying along regularly located curves, with most of the rgy concentrated along the curves corresponding to free and bound waves. This of curves can be described by the equation $D(\omega/n, k/n) = 0$ $(n = 1, 2, 3, ...)$, ere $D(\omega, k) = 0$ is approximated by the linear dispersion relation but does not ıcide with it, especially for large k where there is a tendency for the indicated ation to approximate a straight line. Some other properties of simulated wave ds were also analyzed; these included temporal evolution of the spectra and spatial ribution of the energy of perturbations. The method developed may be applied to road range of problems where the assumption of one-dimensionality is acceptable.

Introduction

mputational techniques for numerical solution of the Navier–Stokes ıations have brought new developments to geophysical fluid dynam- Using modern numerical models, the long-term evolution of several nplicated dynamical phenomena in different fluids, including the at- sphere, can be successfully simulated. However, the long-term simu- ion of a nonlinear multi-mode wave field is difficult to perform, since st numerical schemes for the Euler equations fail to provide sufficient uracy for treating nonlinearities in wave motion. The main source of or is primarily due to the finite difference representation of the vertical ucture of the flow when waves with different wavenumbers are present. us, theoretical and numerical investigations of surface gravity waves usually based on the equations for potential flow with a free surface. this case the flow is fully determined by the form of the surface and the ocity potential on the surface and in its vicinity. The potential mo- n assumption, of course, idealizes the phenomenon, since actual wave tion is both rotational and turbulent. Fortunately, potential theory es many results which agree well with observations. For example, it is l-known that even linear theory yields phase velocity estimates with accuracy of the order of 1%. A much more sophisticated theory, deal- with nonlinear wave–wave interactions (Hasselmann, 1962), which is ɔ based on the potential motion assumption, gives results which are ıfirmed by experimental data.

The main advantage of the potential motion approximation is that the tem of Euler dynamical equations is reduced to the Laplace equation. wever, the solution to the problem of surface wave motion is compli-

cated by
boundar
which is
previous
system (
applicab
on a for
ordinate
Cartesia
coordina
this does
is transfc
must be
culate th
is based c
of the su
was appli
tions (We
for a rela
the conve
wave field

In thi
were simu
Higgins a
(Dold, 19
steepness,
breaking
extremely
expansion
and Sulen
of modula
wave for
methods t

Our g
1-D poten
scales muc
is based on