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Taylor-Galerkin Method For Wind Wave Propagation [1]

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Abstract

Recently Taylor-Galerkin method has been used to successfully solve advection-dominated flow problems. This method performs quite well in reducing numerical dissipation and numerical dispersion of model solutions. In this paper we explore feasibility of applying this method to wind wave propagation.

Introduction

In recent decades wind wave modeling has developed significantly from the empirical approaches of Sverdrup and Munk (1947) and Bretschneider (1958) for example , to spectral approaches including directionality using the radiative transfer equation (e.g. SWAMP Group 1985). At present, the most advanced directionally spectral model is the so-called third generation wave model of which the WAM model is an example (WAMDI Group, 1988). Although these directionally spectral models achieve a significant increase in reliability and computational complexity in wind wave prediction, many uncertainties still remain. Wind waves result from the interaction of several physical processes, specifically propagation, refraction, and source functions. The source functions include atmospheric generation, wave-wave interaction, wave-current interaction, and dissipation. Some of those physical processes can be described with adequate precision, but others like atmospheric generation and dissipation require empirical correction to fit field data. Thus it becomes imperative that numerical errors in model solutions should be not large enough to be attributable to physical processes. Numerical errors in wave models are mainly due to numerical dissipation and numerical dispersion which are associated with the numerical scheme used for wave propagation. At present, no numerical scheme in a fixed grid system can completely avoid this kind of error. In this regard, the WAM model uses the upwind numerical scheme for propagation. This scheme produces significant numerical dissipation and results in a misinterpretation of the physical processes represented by the model. (Tolman 1991)

In this paper we explore the feasibility of applying the Taylor-Galerkin

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method to wave propagation in wave modeling. Recently, the Taylor-Galerkin method has been used to successfully solve advection-dominated flow problems (Donea (1984), Löhner, et al (1984), Peraire, et al (1986), Baker and Kim (1987), Lee, et al (1987)). In this method, a temporal discretization precedes the spatial one, which in essence is considered along the characteristics to achieve a self-adjoint form. This method performs quite well in reducing numerical dissipation and numerical dispersion. In addition, the method employs finite element discretization; this is particularly useful in dealing with a flexible grid system.

Governing Equation and Numerical Solution

The governing equation for the energy density spectrum $F(x,y,t,f,\theta)$ can be simply written as

$$\frac{\partial F}{\partial t}+\frac{\partial c_x F}{\partial x}+\frac{\partial c_y F}{\partial y}+\frac{\partial c_\theta F}{\partial \theta}=S \qquad (1)$$

where (x,y) are the horizontal spatial coordinates, t is time, f is the frequency, θ is the direction, and (c_x,c_y,c_θ) are the propagation velocities in the corresponding coordinates. S is the source term consisting of atmospheric input, nonlinear wave-wave interaction, and dissipation due to whitecapping and bottom friction. Empirical formulations of the physical processes of the atmospheric input and dissipation are one source of model inaccuracy and the discrete interaction approximation for calculating wave-wave interaction is another source of model inaccuracy; these inaccuracies have little to do with the numerical scheme used for propagation. Therefore, we may set the source term $S=0$ in the following numerical examples.

In the numerical solution of (1), we consider splitting the solution into two modes; first mode corresponds to solving (1) without its fourth term, followed by second mode to solving (1) without its second and third terms. Hence, in the following numerical formulations we only consider the first mode equation and, in a numerical sense, the second mode equation is a one dimensional case of the first mode formulation. The Taylor-Galerkin method is employed to numerically solve the first mode equation. In this method a temporal discretization up to $O(\Delta t^2)$ is followed by a finite element method for spatial discretization.

$$\int_\Omega \Delta F N_i = -\Delta t\int_\Omega \nabla\cdot Q N_i - \frac{\Delta t^2}{2}\int_\Omega \nabla\cdot Q\left(c_x\frac{\partial N_i}{\partial x}+c_y\frac{\partial N_i}{\partial y}\right)+\frac{\Delta t^2}{2}\int_{\partial\Omega}\nabla\cdot Q c_n N_i \qquad (2)$$

where the N_i are the shape functions, $Q=(c_xF,c_yF)$, and c_n represents propagation velocity normal to $\partial\Omega$. For computational efficiency, a lumped mass procedure is further invoked. We rewrite (2) as $M\Delta F=q^n$, where M is a mass matrix and the superscript n is the time step. We use the lumped form of M_l and obtain the solution by a simple iteration according to the following equation,

$$M_l\,\Delta F^k = q^n-(M-M_l)\,\Delta F^{k-1} \qquad (3)$$

where the superscript k is the iteration step. Two or three iterations for (3) is generally sufficient for an acceptable solution.

Results

As a first example we study one dimensional propagation of an initial wave field $F=exp(-\frac{(x/20)^2}{2})$. Figure 1 illustrates the result for the Courant number $C_r=0.25$ and the number of iterations $k=2$ for (3). It indicates that the numerical dissipation and numerical dispersion start to show up only at time step $n=200$. The second example is a two-dimensional rotating propagation of a cone-shaped wave field $F=exp(-\frac{x^2+y^2}{800})$. Figure 2 illustrates the result after one complete rotation at $n=200$ for the maximum $C_r=0.2513$ and $k=2$. It has reduced the the peak only about 5 percent with no phase shift. Numerical dispersion is barely discernible in the near field. The third example is for wave field propagation in the θ coordinate only. The initial mean direction is $\theta_m=60°$ and the initial distribution is $cos^2(\theta-\theta_m)$. The maximum $C_r=0.38$ and the directional increment $\Delta\theta=15°$. The results are shown in Figures 3 and 4 and are in agreement with semi-analytical solution. Detailed information for this example will be presented at the conference. More tests are still underway and well be reported at the conference.

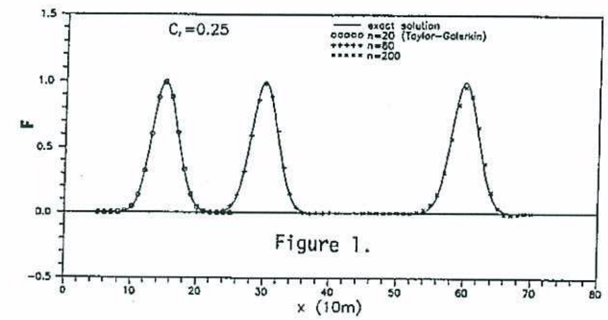
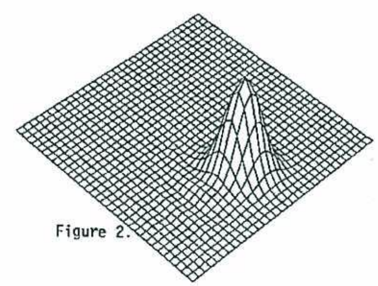


Figure 1.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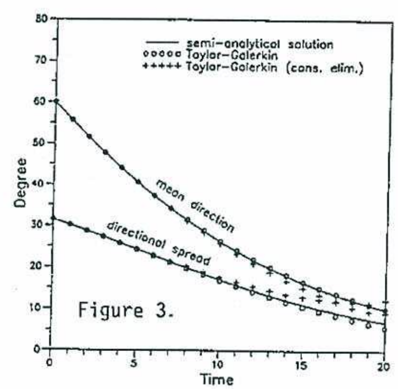

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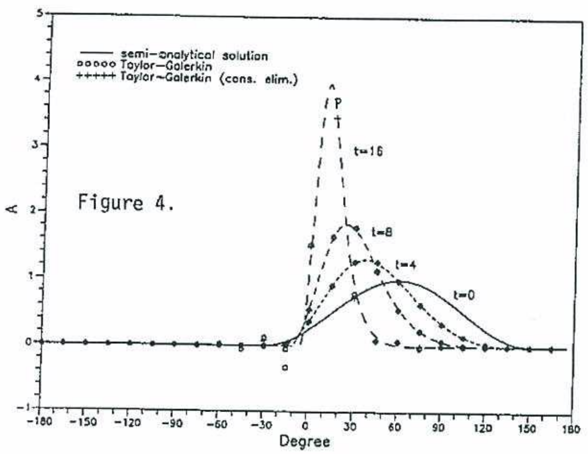

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