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Technical Note

A note on stationary wave modeling with WAVEWATCH III ® †.

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Wind wave modeling at sea and near the coast generally considers the evolution of the wind wave spectrum F in space and time. Away from the surf zone, only the amplitude of the spectral components is considered, and a random spectral phase is assumed[1]. Starting with Gelci et al. (1956, 1957) numerical wave models predict the evolution of such a spectrum, generally using some form of the basic balance equation as formally established by Hasselmann (1960)

$$\frac{\partial F(f,\theta;\boldsymbol{x},t)}{\partial t} + \nabla \cdot \boldsymbol{c} F(f,\theta;\boldsymbol{x},t) = \sum S(f,\theta;\boldsymbol{k},t) \quad , \tag{1}$$

where f and θ are a spectral frequency and direction, respectively (or alternatively the wavenumber k and/or wavenumber vector $\boldsymbol{k}$), $\boldsymbol{x}$ and t are physical space and time, respectively, $\boldsymbol{c}$ represents the characteristic velocities in physical and spectral space (possibly accounting for mean currents), and $\sum S$ represent major sources and sinks of spectral energy. For the present study, only the mathematical properties of the equation are relevant. Equation (1) is a hyperbolic equation that is generally solved by describing the problem with discretized spectra (i.e., spectra with discretized f and θ) on a discretized spatial grid, while marching the solution forward in time. Due to the multi-dimensional nature of the problem, fractional step approaches (Yanenko, 1971) with explicit finite difference schemes are used by virtually all wave models.

The above approach is effective in the deep ocean and in larger water bodies, as the corresponding transient nature of the wave field solution requires the evaluation of the wave spectra in time. When smaller areas with higher spatial resolution are considered, marching a solution forward in time becomes increasingly expensive, as a reduction of spatial grid sizes by a factor β generally increases computational efforts by a factor β^3 if identical grid coverage and spectral resolution are maintained. In small-scale coastal areas, this can make models based on Eq. (1) prohibitively expensive. In such areas, however, information travels rapidly through the entire grid due to the limited spatial extend of the grid. If associated time scales are smaller than time scales at which environmental forcing and boundary data change, a quasi-stationary approach can be used where Eq. (1) reduces to

$$\nabla \cdot \boldsymbol{c} F(f,\theta;\boldsymbol{x},t) = \sum S(f,\theta;\boldsymbol{x},t) \quad . \tag{2}$$

Equation (2) is an elliptical equation that in principle requires massive matrix inversions to achieve a solution. In practice, this matrix inversion is either simplified by considering “half-plane” solutions only, or requires some iterative solution for generally applicable quasi-stationary models. The iterations are typically controlled by dynamic convergence criteria, and/or by a maximum number of iterations allowed. A prominent example of a latter general purpose model is the

[1] Phase resolving o time domain models as used in the surf zone are not considered here.

SWAN model (Booij et al., 1999; Ris et al., 1999). The fact that features of both type of equations are needed in practical wave models can be seen in the fact that the SWAN model was extended for use at larger scale areas by re-introducing the time derivative in Eq. (2), while retaining the iterative solution technique for elliptical equations (e.g., Fraza, 1998; Holthuijsen, 2007, Section 9.5.2).

Quasi-stationary solutions can also be obtained from Eq. (1) by integrating the model until stationarity is achieved. Clearly, such a computation is equivalent to iterating toward a solution of Eq. (2), and in fact similar convergence criteria can be used. An advantage of using this approach for the hyperbolic equations is that the time for wave information to travel through the grid can be estimated explicitly from the characteristic velocities of the dominant waves, with the caveat that this will not be an 'exact science' due to the nonlinear interactions between all scales in the spectrum. If the transient time of dominant information through the grid is estimated as t_t, the time scale for reaching stationarity t_s becomes

$$t_s = \alpha t_t \quad , \tag{3}$$

where α is a relatively small number larger than 1. With a numerical time step Δt_n in the hyperbolic model, the number of iterations n_i then simply becomes.

$$n_i = t_s / \Delta t_n \quad . \tag{4}$$

Such a fixed number of iterations may not be the most effective way to reach stationarity, and may limit accuracy of the final solution, but is a benefit for operational models as the model run time becomes predictable.

This time scale can also be used to determine if a quasi-stationary approach should be used. Assuming that an analysis of time scales of forcing and required output frequencies dictates that model output should be available at solution intervals Δt_s, quasi-stationary solutions can only expected to be valid if

$$t_s \ll \Delta t_s \quad , \tag{5}$$

and otherwise a non-stationary approach will be required.

A full time-marching solution of the hyperbolic equations requires

$$n_m = \Delta t_s / \Delta t_n \quad . \tag{6}$$

numerical time steps to reach the next solution. In quasi-stationary conditions where Eq. (5) is satisfied, $n_m > n_i$, and tentatively a reasonably accurate quasi-stationary solution for the next output time can be obtained at a factor γ less computational effort where

$$\gamma = \frac{n_m}{n_i} = \frac{\Delta t_s}{t_s} \quad . \tag{7}$$

In the context of the hyperbolic equations, a set of quasi-stationary solution can then be achieved by using a simple "time compression" approach, where for each time step the spectral increments are computed as

$$\Delta F(f,\theta) \propto \Delta t_n \sum S(f,\theta) \quad , \tag{8}$$

but where the marching of physical time is accelerated by a factor γ, i.e.,

$$t_i = t_{i-1} + \gamma \Delta t_n \quad , \tag{9}$$

where t_i and t_{i-1} are the present and previous discrete times, respectively. Equation (9) represents linear time compression. Alternatively, nonlinear compression schemes can be devised, where γ represents the mean time acceleration per output time step Δt_s. Finally, discontinuous time compression can be performed by discontinuously incrementing the time before or after the dynamic calculations so that the time the end of the computation interval s consistent with the time increment Δt_s, for instance

$$t_i = t_0 + (n_m - n_i + i)\Delta t_n \quad , \tag{10}$$

where t_0 is the time at the beginning of the time interval Δt_s, and t_i represents the time after the i^{th} of n_i actually applied time steps.

If a set of nested grids is considered, this time compression with the acceleration factor γ can be applied to all grids considered with the additional refinement that γ by definition needs to be larger than 1, replacing Eq. (7) with

$$\gamma = \max\left(1.\,, \, \frac{\Delta t_s}{t_s}\right) \quad . \tag{11}$$

This creates a natural transition from fully unsteady to quasi-steady approaches, which becomes particularly elegant in the two-way nested or mosaic approach to wave modeling as implemented in WAVEWATCH III ® (Tolman, 2008). This will, however, require a subtle modification to the algorithm used to dynamically determine time stepping in the individual grids (Tolman, 2008, Table 1). in the original mosaic implementation, time stepping and synchronization between grids is governed by the overall numerical time step associated with each grid. For (quasi-) stationary grids, this time step has to be replaced by Δt_s as used above, and thus Δt_s has to become an effective minimum grid synchronization time interval.

One complication occurs when using a definition of γ compatible with Eq. (11) for grids with prescribed boundary conditions (nesting). In traditional (non-stationary) nesting, boundary data needs to be updated at the end of the time step to be consistent with numerical finite difference schemes used in the model. In the quasi-steady approach, however, boundary data needs to be consistent throughout the computation with the data corresponding to the time for which the solution is to be obtained. In the context of time marching using the hyperbolic equation, the latter will be the boundary data at the *end* of the time step. Note that the same considerations apply to all external model inputs (wind, currents, water levels and ice).

In a typical wave model, the treatment of boundary and other input data is governed by the model time, whereas the actual model time has no direct impact on the model integration according to Eq. (8). Hence, proper treatment of boundary and input data ranging from full time integration to quasi-stationary approaches can, in principle, be controlled by a proper time compression scheme replacing Eq. (9) with a nonlinear description with n_i integration steps for Eq, (8). Qualitatively similar results are obtained with the discontinuous time compression scheme (10), of which the degenerate form

$$t_i = t_0 + n_m \Delta t_n = t_0 + \Delta t_s \quad , \tag{12}$$

will result in wave model behavior consistent with the fully stationary approach.

It is fairly trivial to implement the above time compression approach in the WAVEWATCH III model. A time scale t_s needs to be attributed to each grid, and all time compression can be easily added in the main wave model routine W3WAVE in the file w3wavemd.ftn. Minor modifications to the mosaic wave model routine WMWAVE in the file wmwavemd.ftn by introducing a minimum synchronization time $\Delta t_{s\,\mathrm{min}}$ interval as described above. The main effort will be required for finding suitable time compression formulations that also properly address the use of boundary and forcing data, as well as defining guidelines for using non-stationary and stationary grids in a single mosaic.

With such a time compression algorithm, the main attention for obtaining accurate yet economical model results will be required while setting up models. The mosaic approach in WAVEWATCH III tentatively provides a unique way of doing this systematically by first running the wave model in fully non-stationary mode ($t_s = \infty$ for all grids, $\Delta_{s,\mathrm{min}} = 0$), and then systematically reducing t_s and increasing $\Delta_{s,\mathrm{min}}$ to realistic values until differences in results with the fully non-stationary model become unacceptable. Alternatively, the model can first be set up in a cheap way (small t_s, large $\Delta t_{s,\mathrm{min}}$), after which the model is made incrementally more unsteady until no significant changes is results are seen. This will make the development of mosaics more complicated, but the resulting models better understood.

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