Operational Implementation of a Multi-grid Wave Forecasting System*

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1 Introduction

NCEP provides guidance for wind waves and ocean swells using the third generation spectral wave model WAVEWATCH III (Tolman, 2002c). The model uses a third order propagation scheme that minimizes numerical diffusion and a parallel code was developed to increase computational speed (Tolman, 2002a).

The current forecast setup uses WAVEWATCH III v2.22 and consists of a suite of models and includes a global forecast model (NWW3 – Tolman et al. 2002), three regional models for the Alaskan Waters (AKW – Chao et al. 2003a), Eastern North Pacific (ENP – Chao et al. 2003b) and Western North Atlantic (WNA – Chao et al. 2003c), and two operational hurricane models (Chao and Tolman, 2000, 2001; Alves et al., 2005) for the Northern Atlantic (NAH) and Northern Pacific (NPH). Boundary conditions for the regional models are obtained from the global model. The global model is also used to drive an ensemble global ocean wave forecast system (EGOWaFS – Chen 2006). Apart from that there is also a Great Lakes forecast model (GLW) which is run separately and therefore out of the present scope.

Validation studies (Tolman, 2002b,d; Tolman et al., 2002) have shown that the suite of operational models predict the ocean state well. The models were further improved to account for the blocking effects of unresolved islands with the help of obstruction grids (Tolman, 2003). However, computational costs limit the global operational model to a grid of $1.25^\circ \times 1^\circ$, and the regional models to a $15' \times 15'$ grid. While this resolution is adequate for providing guidance to regional forecasts, coastal forecasts (put out by the various Weather Forecast Offices) are needed at grids with resolutions of 5km or higher. Furthermore, boundary conditions for the regional grids are obtained from the global model, but the higher resolution effects are not passed back from the regional grids to the global grid.

A new multi-grid version of WAVEWATCH III has been developed (Tolman, 2006, 2007a,b) that has several technological upgrades and a few improvements to the physics as well. To make full use of these upgrades in the operational forecasts, a new forecast model has been developed using multi-grid WAVEWATCH III. The new operational model has been designed to increase the spatial resolution of the guidance forecasts while optimizing the operational costs. It also improves on the delivery times of the output products over the existing suite of models and provides additional summary information on the spectral state of the ocean.

2 WAVEWATCH III - v3.1x A brief overview

Many significant changes have been introduced between version 2 and version 3 of WAVEWATCH III.

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Most of these changes have been technological upgrades and a detailed discussion on the new features of WAVEWATCH III can be found in Tolman (2006, 2007a,b). For completeness a brief overview of the new features is provided here.

Mosaic approach to wave modeling

Instead of representing the computational domain by a single grid as was done in the past, the new version of WAVEWATCH III now allows the domain to be represented by an arbitrary number of grids of different resolution. Grids are ranked according to their resolution, with the lower resolution grids having a lower rank, and grids with similar resolution having the same rank. Each grid acts as a separate wave model, however there is full two-way interaction between the grids. Data for the boundary points of the higher ranked grids are obtained from the lower ranked grids that these points lie in. Similarly in lower ranked grids, points that lie inside the domain of higher ranked grids receive spatially averaged data from the latter. This allows sea states that develop in high resolution areas to propagate out to lower resolution areas. Overlapping grids with the same rank are reconciled in their area of overlap. This mosaic approach to wave modeling allows us to use a single model driver to generate forecasts at several different resolutions over the computational domain, thus, greatly simplifying the operational requirements.

The boundary points (where data is passed from the lower to higher ranked grids) can now be defined at the edge of the grid, precluding the need for spurious land values along the edge as was the practice in WAVEWATCH III v 2.22 (Tolman, 2002c). The new model also allows boundary points to be defined inside the grid, making a distinction between land, active and excluded points. This feature has been heavily used in developing an optimal forecast model (section 3).

Partitioning of Wave Spectra

An algorithm developed by Hanson and Jenson (2004) to partition the energy spectrum has been added to the model. It is based on a digital image processing watershed algorithm (Vincent and Soille, 1991) and divides the energy spectrum into partitions that represent energy from sub-peaks in the spectrum. Each partition represents a sea state. Quantitative features of the partitioned spectra (e.g. significant wave height, peak period) as well as the number of partitions are now part of model output. Examples of partitioned output can be found in Tolman (2007a).

New Physics

Two new source terms have been added to the model to better simulate physical processes. One is the linear growth parametrization of Cavaleri and Malanotte-Rizzoli (1981) together with a filter for low frequency energy (Tolman, 1992). This term has been added to consistently spin up the model from quiescent conditions and to better simulate the initial wave growth. The second source term is a depth induced breaking term from Battjes and Janssen (1978) that simulates energy dissipation due to surf zone breaking. This term has been added to extend the applicability of the model into shallow water environments. The source term is based on the criterion that all wave heights exceeding a maximum height in the surf zone will break and the energy dissipation rate is based on dissipation of a turbulent bore.

Obstruction Grids

The algorithm for using obstruction grids to simulate blocking effects from unresolved islands (Tolman, 2003) has been an integral part of WAVEWATCH III since 2002 and remains unchanged in the new version. However, the approach used in building these obstruction grids has changed. Even though this is not a part of WAVEWATCH III itself it is mentioned here because of the role it plays in the operational modeling suite. Obstruction grids for the current suite of operational wave models (NWW3, AKW, WNA, ENP, NAH and NPH) were constructed manually. A new grid generation package (Chawla and Tolman, 2007, 2008) has been developed for WAVEWATCH III to automate the grid generation process and create consistent grids across different resolutions. In this package obstruction grids are generated using the GSHHS shoreline database. The GSHHS database (Wessel and Smith, 1996) contains an exhaustive global list of shoreline boundaries (over 180000) and covers land bodies ranging from small atolls to large continents.

3 Multi - grid forecast model (NMWW3)

A single operational multi-grid forecast model (hereby referred to as NMWW3) has been designed to replace the existing suite of operational models. The new forecast model has been designed to run on the same schedule as the old models, i.e. 4 forecast cycles a day (at 0Z, 06Z, 12Z and 18Z). Each cycle consists of a 189 hour long run with 9 hours of hindcast and 180 hours of forecast. Restart files are generated at the 6 hour time step to be used as starting conditions for the next cycle.

3.1 Grids

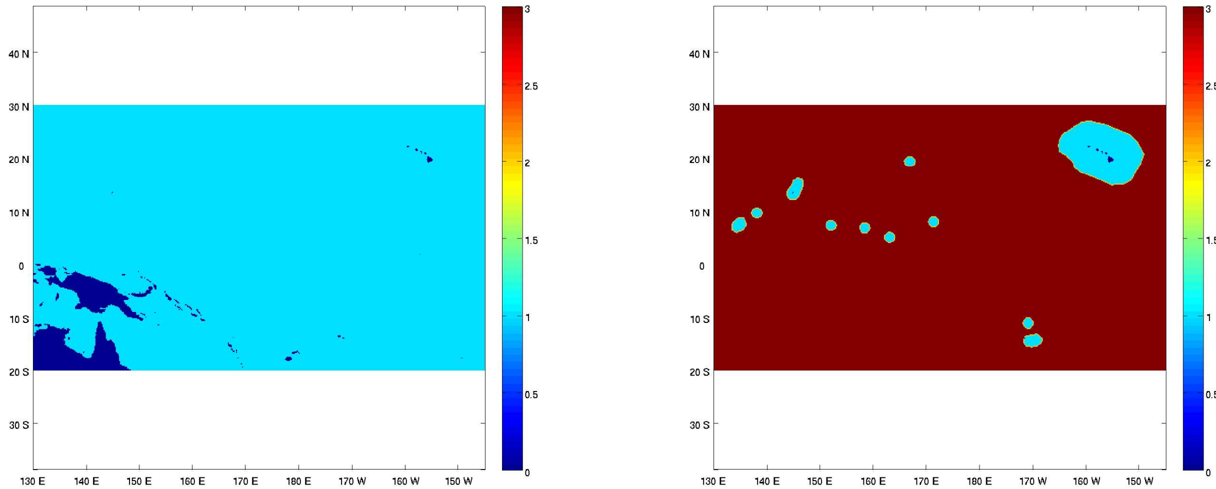
The main aim of developing the NMWW3 model was to provide guidance forecasts for the WFOs (Weather Forecast Offices) and the regional prediction centers (Ocean Prediction Center and Tropical Prediction Center) at a suitable resolution within the available computational constraints. The WFOs provide forecasts for the US coastal waters on grids with a 5km or finer resolution. These grids extend out to approximately 60 nautical miles along the US west coast and 40 nautical miles along the US east coast. The regional forecasts on the other hand are provided on $10'$ resolution grids by the two prediction centers. The regions that fall under the mandate of regional forecasts include the US west and east coasts, Alaska, Hawaii, Gulf of Mexico and some of the islands in the South Pacific.

To provide appropriate guidance forecasts, the global domain was divided into eight grids – one global $30' \times 30'$ grid, four regional grids ($10' \times 10'$ grids for the US East Coast including the Gulf of Mexico and the Caribbean Sea, for the US West Coast and for the Pacific Ocean including Hawaii and select islands in the Eastern Pacific, and a $15' \times 10'$ grid for Alaska), and three coastal grids ($4' \times 4'$ grids for the US East Coast including the coastal waters of Puerto Rico and for the US West Coast including the coastal waters of Hawaii, and an $8' \times 4'$ grid for Alaska).

Optimal high resolution regional grids were designed by taking advantage of the model's flexibility in assigning boundary points inside a grid (section 2). This is highlighted by the mask generated for the Pacific Ocean $10' \times 10'$ grid (Fig. 1). This grid covers Hawaii and other smaller islands in the Pacific Ocean where higher resolution results are desired. In the new model, points where high resolution results are not needed can be excluded. This significantly reduces the number of active points and the subsequent computational cost. Fig. 2 shows the maximum resolution available from the eight grids. The global domain excludes the polar ice cap regions and extends from -77.5°S to 77.5°N and 180°W to 180°E. Separation of the computational domain into eight different grids was done to optimize the parallel implementation within the constraints of desired regional grid resolutions. Note that the lower resolution grids extend to the coast, creating a full overlap with higher resolution grids. Fig. 3 shows a snapshot of wave heights at the three different resolutions individually as well as the composite wave heights for the 120 hour forecast for the 12Z cycle on Oct 26$^{\text{th}}$, 2007. The composite image was generated by plotting results for all grids consecutively (ordered from low to high resolution) and shows the seamless distribution of wave heights across the grids.

3.2 Forcings

The wave model is forced by the 10 meter winds, sea surface temperature and ice data. In the new multi-grid model the individual grids can obtain the external forcings from either a single grid that covers the entire domain (data is interpolated or averaged from the forcing grid to the computational grid internally in the code) or have unique forcing data sets for each grid. The former approach is used in NMWW3 as it precludes the need to develop individual forcing files for each grid as well as maximizes consistency of winds across the different grids. Sea ice concentration data are updated once a day over a global $5' \times 5'$ grid using an automated passive microwave analysis (Grumbine, 1996). Forecast winds are obtained from the Global Forecast System (GFS – previously known as AVN/MRF) (Kanamitsu, 1989; Kanamitsu et al., 1991; Caplan et al., 1997). Hindcast winds are obtained from the Global Data Assimilation System (GDAS) (Kanamitsu, 1989; Derber et al., 1991) and uses global observations to provide initial conditions for the GFS model. GDAS provides the best estimate of the winds and is the primary reason why we run 9 hours of hindcast simulation.



(a) Original Mask

(b) Final Mask

Figure 1: Original and final masks for the Pacific $10' \times 10'$ grid. Mask values of 0, 1, 2 and 3 correspond to land, active, boundary and inactive points respectively. Active points are reduced significantly from the original (147426) to the final (5873) mask.

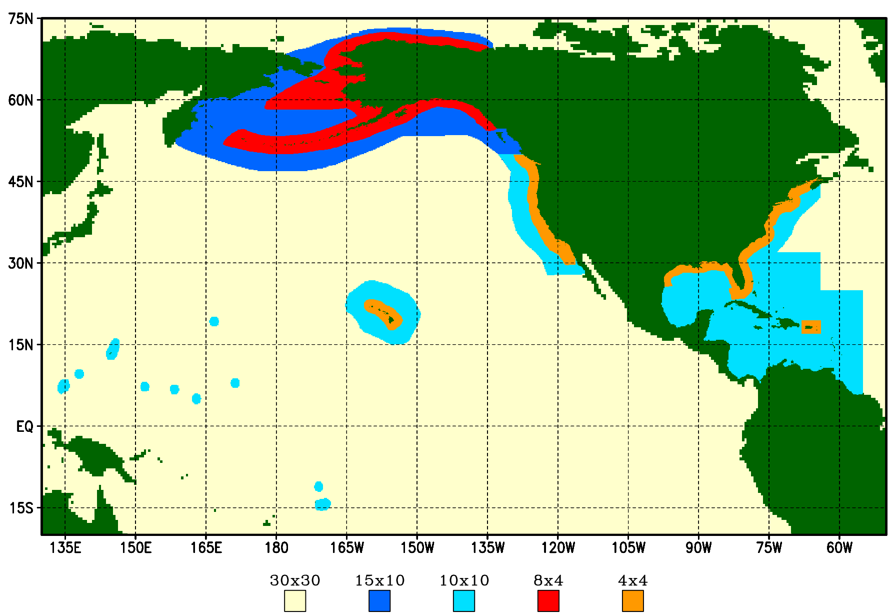
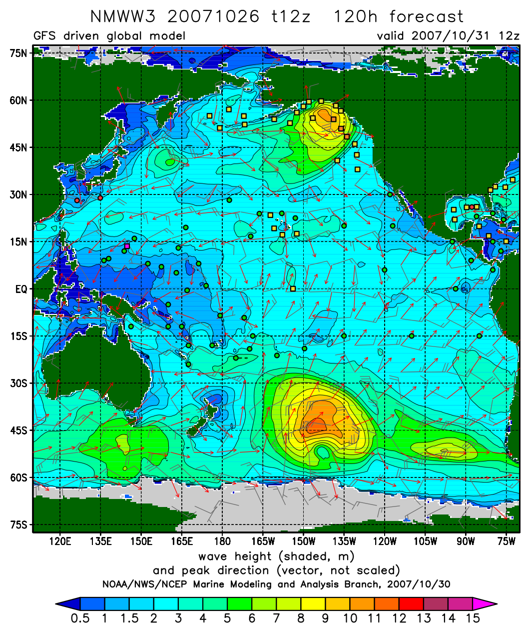
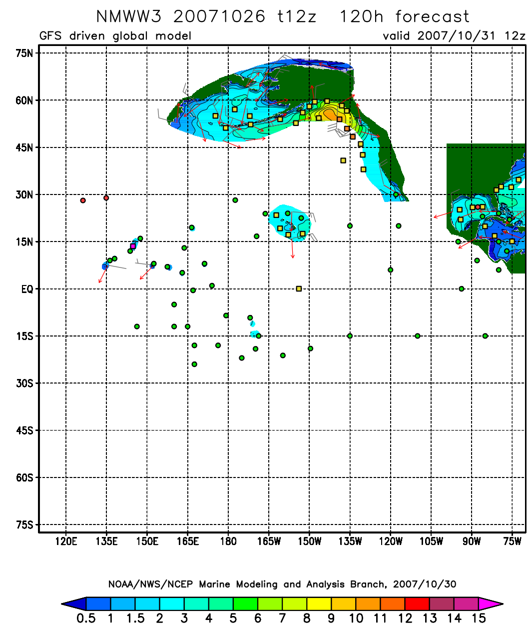


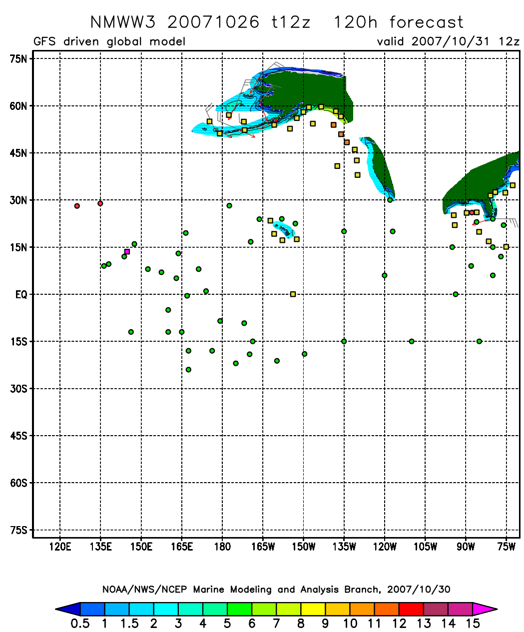
Figure 2: Grids for NMWW3. Grid resolution given in minutes.



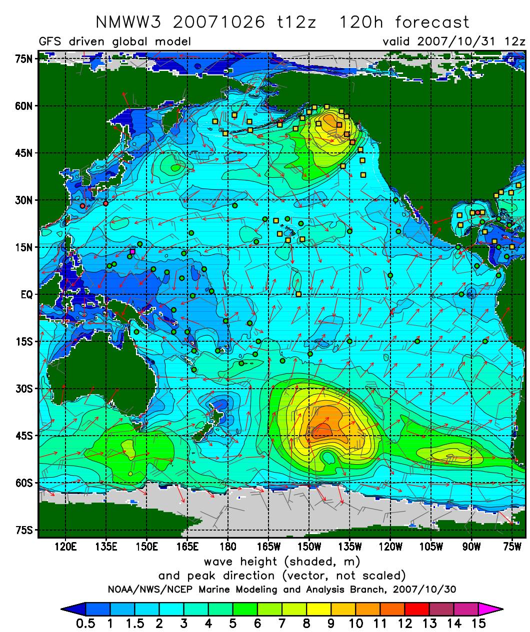
(a) Global Grid



(b) Regional Grids



(c) Coastal Grids



(d) Composite

Figure 3: Snapshot of wave height distribution at the different grid resolutions

Apart from the hindcast winds GDAS is also used to determine the initial sea surface temperature. All the wind data are provided on a 3h time step.

Like its predecessors, NMWW3 was initially designed to be run after all the forcing files were ready. This meant that a particular cycle could only be launched after the GFS files for the 180 hour forecast of that cycle were made available. To minimize this delay, NMWW3 was redesigned so that it could run side by side with the GFS model. A time stamp file is used to determine the available wind data and to control the forward propagation of the model. When new wind data becomes available this file is updated and the model allowed to propagate forward in time. This way, the model run is now completed within a few minutes of the GFS 180h wind file being made available, significantly improving the delivery time of forecast products. This approach has been feasible in NMWW3 because the computations are driven by a single model, and due to the highly modular code design consistent with the Earth System Modeling Framework (ESMF) concepts.

3.3 Products

There are two types of output data in NMWW3 − field data and point data. This is similar to the output fields of the current operational wave models. However, there are some distinct differences.

Field data consist of the mean characteristics of the ocean spectral data represented on a spatial domain. The same field outputs that are part of the current operational models are also available in the new model. The new model also includes spectral characteristics (significant wave height, peak period and mean direction corresponding to peak period) of the partitioned spectra.

The wind wave portion of the spectrum is given by a single component, even if the spectrum in this region has multiple peaks. For the partitioned output the wind wave fraction is given by

$$W = E^{-1} E|_{Up>c} \tag{1}$$

where E is the total energy, $E|_{Up>c}$ is the portion of the energy spectrum directly under the influence of the winds and corresponds to part of the spectrum where the projected wind speed U_p is greater than the phase speed $c = \sigma/k$. The projected wind speed is given by

$$U_p = C_m U_{10} \cos(\theta - \theta_w) \tag{2}$$

where U_{10} is the 10 meter wind, θ and θ_w are the directions of the waves and wind respectively, and C_m is a constant multiplier that allows for moving the wind-swell boundaries to lower frequencies.

The partitioning algorithm does not put a limit on the number of possible swell fields in the spectrum. However, in the output we currently limit ourselves to the first two swell fields (based on local wave height partition) which are referred to as the primary and secondary swells.

Field data is packed using the GRIB2 standard from World Meteorological Organization (WMO) which provides greater flexibility in meta data handling and more efficient packing options than the earlier GRIB standard (which is used in the older modeling suites). To allow for future increases in the number of output swell fields, swell parameters are stored as vertical levels, with the first level corresponding to the primary swell.

Like the current forecast model suites, point output data in NMWW3 provides both detailed spectral data as well as the mean spectral characteristics. As of now the mean spectral characteristics for the point outputs are not obtained using the partitioning algorithm but that is expected to change in the near future. Since in NMWW3 a point location can be in multiple grids, the data are retrieved from the highest rank grid (highest resolution) that the point lies in. As before, the list of points include current and old buoy locations, virtual buoy locations as well as boundary locations (for collaboration projects within and outside NOAA using NMWW3 results as boundary conditions for external models). This list has been expanded to include buoy points from NDBC (National Data Buoy Center), ENCAN (Environment Canada), GOMOOS (Gulf of Maine OOS), IDT (Irish Department of Transport), UKMO (UK Met Office), SHOM (Service Hydrographique et Oceanographique de la Marine) and METFR (Meteo France) among others.

Apart from data output, the Marine Modeling and Analysis Branch also maintains a web page (http://polar.ncep.noaa.gov/waves/) with access to the last six forecast cycles. Access is available to both the output data as well as images. Since the two way coupling of the multi-grid model provides seamless data across grids (Fig. 3), display views are not lim-

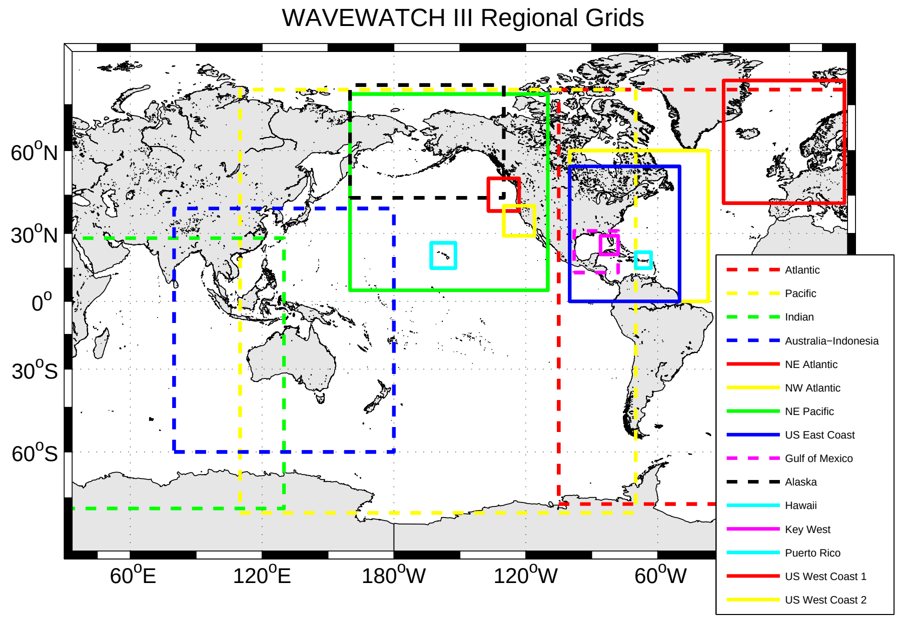


Figure 4: Summary of the different display views available for NMWW3. Images can be accessed from http://polar.ncep.noaa.gov/waves/

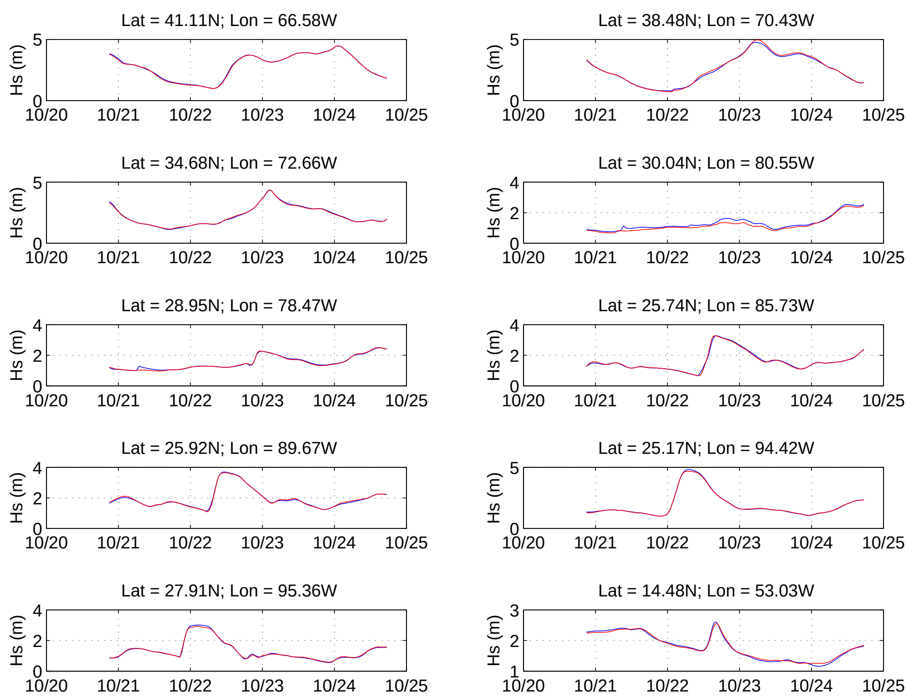


Figure 5: Significant wave heights from the WNA and NMWW3 models at ten different locations in the Atlantic Ocean and Gulf of Mexico. Blue lines correspond to the WNA model and red line to the NMWW3 model. The x axis corresponds to time in mm/dd format and the y axis is the significant wave height in meters. Nowcast time for this run was 10/21/2007 0000 hours.

ited to the actual grids. Currently images with 15 different views are generated. Fig 4 shows the different available views.

4 Comparisons

Most changes introduced in NMWW3 have been technical upgrades (with the exception of the linear growth term and shallow water breaking). As such these changes should not have a significant impact on forecasts in the open ocean. This is borne out by comparisons between the current regional Atlantic Ocean model (WNA) and NMWW3 in the Atlantic Ocean and Gulf of Mexico at several locations (Fig. 5). These points are in deep enough water that depth induced breaking is not a factor here. As such the minor differences between the two models is probably due to the linear growth term.

The biggest impact of the upgrades in NMWW3 is on waves driven by land falling hurricanes. For these cases the higher resolution near the coasts and depth induced breaking terms are both expected to have a significant impact. To quantify this impact, both WNA and NMWW3 models were used to simulate two recent hurricanes in the Atlantic − hurricane Katrina in the Gulf of Mexico (Fig. 6) and hurricane Isabel along the US East Coast (figure not shown).

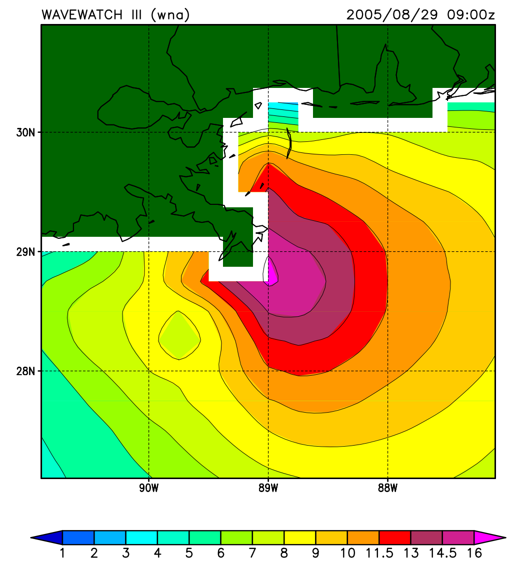
To isolate the impact of depth induced breaking in hurricane Katrina, NMWW3 results have been shown both with and without surf zone physics. Even without the surf zone physics (Fig. 6b) the impact of high resolution coast lines can be clearly seen in the NMWW3 results. The WNA model does a poor job of resolving the coast line in comparison with the NMWW3 model, particularly around the Mississippi river delta, Breton Sound and Lake Borgne. The obstruction effects of the Chandeleur islands are also absent since these islands are absent in the obstruction grids for the WNA model, leading to significantly larger waves in the Breton and Chandeleur Sounds. The Chandeleur islands being part of the GSHHS database are automatically taken into account when the obstruction grids for the NMWW3 model were generated and the sheltering effect of these islands can be seen in the results (Fig. 6b). Adding surf zone physics also removes the bulls-eye patterns in wave heights seen near the mouth of the Mississippi river delta as well as dissipates wave energy in the shallow waters. Wave height to depth ratio plots (Fig. 7) underscore the importance of having a depth limited breaking source term to realistically estimate wave heights near the coast during this and other similar events. As expected in the deeper open waters both models give very similar results.

5 Conclusions

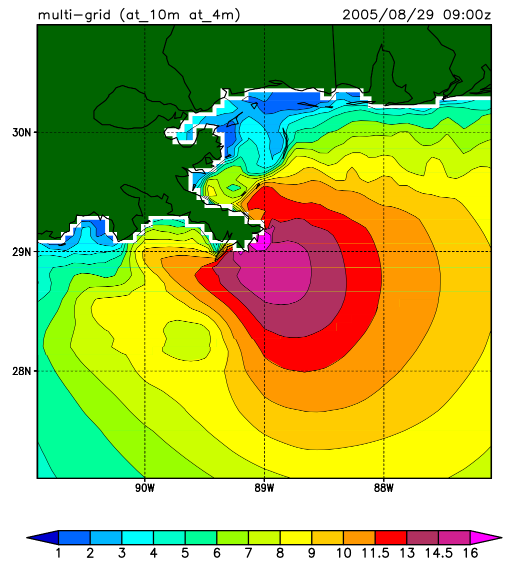
A new operational wave model (NMWW3) has been implemented at NCEP using the multi-grid version of WAVEWATCH III. The model is currently undergoing extensive testing in a simulated production environment. This model replaces the existing suite of operational models (NWW3, AKW, WNA and ENP). Since the NWW3 model is also used in the ensemble model (EGOWaFS) and because data assimilation of the NWW3 code has not been ported to the multi-grid model, the NWW3 model will continue to be run for the foreseeable future.

NMWW3 provides increased spatial resolution (coarsest grid is $30' \times 30'$ and finest grid is $4' \times 4'$, in comparison to $1.25^\circ \times 1^\circ$ and $15' \times 15'$ for the present suite of models) as well as additional output products corresponding to the partitioned spectra. There is also a significant improvement in the delivery time of forecast products. Comparisons with the current operational suite shows that the results in the deeper open waters are comparable. NMWW3 provides more realistic wave fields closer to the coast, particularly for land falling hurricanes. From an operational view point NMWW3 is convenient because it replaces three (soon to be four) models with a single model, making monitoring and maintenance much easier.

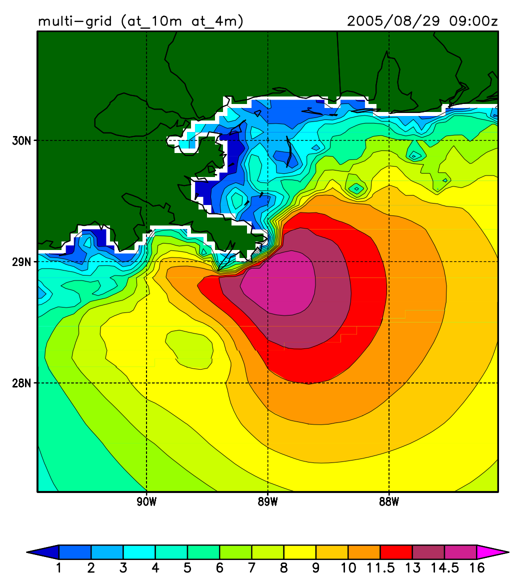
Though the multi-grid version of WAVEWATCH III has led to a considerably improved operational forecast model, the main motivation for developing a multi-grid two-way nested wave model was to improve predictions during hurricanes. In a proof of concept paper Tolman and Alves (2005) showed that due to interpolation errors in wind fields, wave fields generated in a grid moving with the eye of the hurricane can be substantially different from the wave field generated by static grids. With the development of multi-grid WAVEWATCH III it is now feasible to develop hurricane models for waves along the lines of atmospheric hurricane models (Bender et al., 1993; Kurihara et al., 1995). The development of next gen-



(a) WNA model

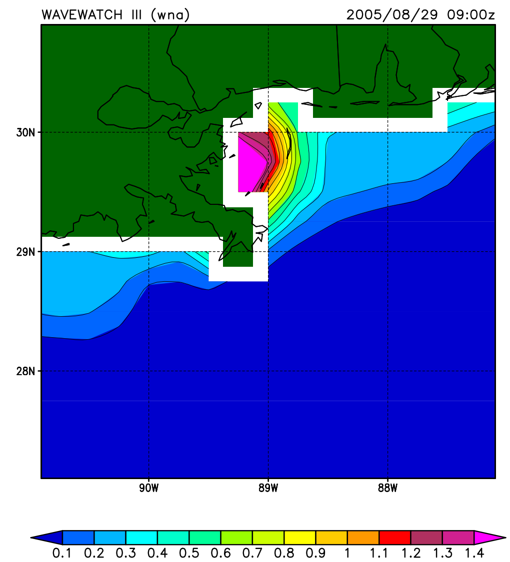


(b) NMWW3 model without breaking

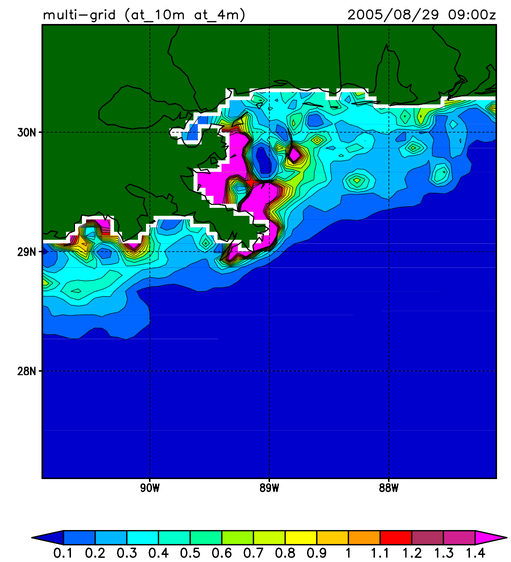


(c) NMWW3 model with breaking

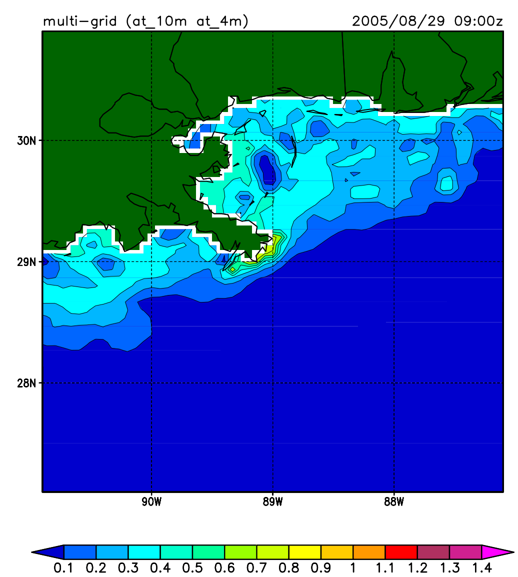
Figure 6: Significant wave heights at land fall during hurricane Katrina



(a) WNA model



(b) NMWW3 model without breaking



(c) NMWW3 model with breaking

Figure 7: Wave height to depth ratio at land fall during hurricane Katrina

eration hurricane models to replace the current NAH and NPH modeling suites is one of our near term goals. This will require that the multi-grid approach is expanded to include relocatable grids.

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