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## Interactive modeling of surface waves and boundary layer <sup>1</sup>

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### Abstract

A new theoretical approach to investigate the nonlinear wave dynamics and wind-wave interaction is developed on a coupled model of wave boundary layer (WBL) and surface waves dynamics. WBL-model is based on the nonstatic Reynolds equations written in nonstationary conformal surface-following coordinate system in the 2-D domain above an arbitrary periodic moving surface which may be represented by a Fourier series. Closure scheme is based on a full turbulent energy evolution equation. Wave dynamics are simulated based of the equations for potential waves. The solutions for air and water components are coupled at each time step by assimilation of surface pressure (obtained from the boundary layer model) into wave model, and shape of the surface and surface velocity components (obtained from the wave model) into boundary layer model. The method developed may be applied to a broad range of wave dynamics and wind-wave interaction problems where the assumption of two-dimensionality is acceptable.

### 1. Introduction

Previous investigations of dynamic interaction between wind and waves were performed mostly for idealized cases of steady monochromatic harmonic waves, which were predescribed at the air-sea interface (Gent, Taylor, 1977; Chalikov, 1978, 1986). However, a recently developed method for numerical solution of potential wave equations (Chalikov, Sheinin, 1997) allows the possibility to simulate the long-term evolution of nonstationary surface wave fields. Wave model is based on the basic equations of potential flow with a

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free surface written in conformal surface-following nonstationary coordinates. This model is coupled with Wave Boundary Layer (WBL) model. The WBL is defined as the lowest part of the atmospheric boundary layer above the surface waves the structure of which is directly influenced by wave-produced fluctuations of velocity and pressure. The exchange of momentum, energy and mass between air and water depends considerably on the specific properties of the WBL. Direct empirical data on the statistical structure of the WBL is sparse, and the only viable method of investigating the WBL is through numerical modeling.

## 2. Coupled model

Let us consider the motion of a two-layer liquid in the domain

$$-\infty < x < \infty, \quad -\infty < y < \infty, \quad -H_w \leq z \leq H_a, \quad (1)$$

with interface  $h(x, y, t)$ . The origin of coordinate  $z$  coincides with the mean level  $h$ . Density of liquid in upper part of the domain is  $\rho = \rho_a$ , and in the lower part is  $\rho = \rho_w$ . The motion of the surface obeys a kinematic boundary condition

$$w_0 = h_t + u_0 h_x + v_0 h_y, \quad (2)$$

where  $u_0$  and  $v_0$  are velocity components at the surface  $z = h$  (subscripts of independent variables denote partial differentiation with respect to the variable.)

A model describing the dynamics of a two-layer liquid is formulated with the following restrictions

1. Surface  $h(x, y, t)$  is periodic in  $x$  and  $y$  directions with period  $2\pi L$ . Consequently, surface may be represented by Fourier expansion

$$\eta(x, y, t) = \sum h_{k,l}(t) \Theta_{kl}(x, y); \quad (3)$$

where  $k = 1, 2, 3, \dots, \infty$  and  $l = 1, 2, 3, \dots, \infty$  are wave numbers in  $x$  and  $y$  directions, and  $\Theta_{kl}$  are the basis functions.

2. Direction of  $x$ -axis coincides with direction of tangential force  $\tau$  applied at upper boundary of domain  $z = H_a$ .  $\tau$  is a constant in space and time.

3. Fourier coefficients for the surface obey conditions  $h_{kl} = 0$  at  $l > M$ . This condition assumes that up to truncation wave number  $k = M$  surface  $h(x, y, t)$  may be considered as a one-dimensional in the horizontal domain.

4. Probability distributions for surface disturbances and turbulent fluctuations of pressure and velocity are invariant with respect to parallel shifts and reflection with respect to  $X$ -axis.

## 2.1 Model of boundary layer above waves

Let us consider the conformal surface-following coordinate transformation for upper domain at  $\zeta > 0$

$$x = \xi - \sum_{-M \leq k \leq M, k \neq 0} \eta_{-k}(\tau) \frac{\cosh k(\tilde{H}_a - \zeta)}{\sinh k\tilde{H}_a} \vartheta_k(\xi), \quad (4)$$

$$z = \zeta + \eta_0(\tau) + \sum_{-M \leq k \leq M, k \neq 0} \eta_k(\tau) \frac{\sinh k(\tilde{H}_a - \zeta)}{\sinh k\tilde{H}_a} \vartheta_k(\xi), \quad (5)$$

where  $\eta_k$  are the coefficients of Fourier expansion of the free surface  $\eta(\xi, \tau)$  with respect to the new horizontal coordinate  $\xi$ , and  $\vartheta_k$  denotes the function

$$\vartheta_k(\xi) = \begin{cases} \cos k\xi & k \geq 0 \\ \sin k\xi & k < 0, \end{cases} \quad (6)$$

(note that  $(\vartheta_k)_\xi = k\vartheta_{-k}$ , and  $\sum (A_k \vartheta_k)_\xi = -\sum k A_{-k} \vartheta_k$ );  $M$  is the truncation wavenumber to be used in numerical integration.

Euler equations after transformation and averaging (Chalikov, 1978) can be expressed as (signs of averaging for first order moments are omitted)

$$\frac{dJu}{d\tau} = -\frac{\partial p x_\xi}{\partial \xi} + \frac{\partial p z_\xi}{\partial \zeta} - \frac{\partial (x_\xi \overline{u'u'} + z_\xi \overline{u'w'})}{\partial \xi} - \frac{\partial (-z_\xi \overline{u'u'} + x_\xi \overline{u'w'})}{\partial \zeta}, \quad (7)$$

$$\frac{dJw}{d\tau} = -\frac{\partial p z_\xi}{\partial \xi} - \frac{\partial p x_\xi}{\partial \zeta} - \frac{\partial (z_\xi \overline{u'w'} + x_\xi \overline{w'w'})}{\partial \xi} - \frac{\partial (-z_\xi \overline{u'w'} + x_\xi \overline{u'w'})}{\partial \zeta}, \quad (8)$$

where  $\frac{d}{dt}$  denotes a total time derivative

$$\frac{dJ(\cdot)}{d\tau} = \frac{\partial J(\cdot)}{\partial \tau} + \frac{\partial J\mathbf{u}(\cdot)}{\partial \xi} + \frac{\partial J\mathbf{w}(\cdot)}{\partial \zeta}, \quad (9)$$

The continuity equation becomes

$$\frac{\partial J}{\partial \tau} + \frac{\partial J\mathbf{u}}{\partial \xi} + \frac{\partial J\mathbf{w}}{\partial \zeta} = 0. \quad (10)$$

Equations (7)-(8) are written in nondimensional form, with the following scales: length  $L$ , time  $\mathcal{T} = L^{1/2}g^{-1/2}$ , velocity  $L^{1/2}g^{1/2}$ , pressure  $\mathcal{P} = \rho_a g L$  ( $g$  is acceleration of gravity).

In equations (7-10),  $J$  is a Jacobian of the transformation given by (4),(5):

$$J = x_\xi^2 + z_\xi^2 = x_\zeta^2 + z_\zeta^2, \quad (11)$$