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## Chapter 7

### Direct modeling of one-dimensional nonlinear potential waves

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#### Abstract

A method for numerical investigation of nonlinear wave dynamics based on direct hydrodynamical modeling of 1-D potential periodic surface waves is presented. By a nonstationary conformal mapping, the principal equations are rewritten in a surface-following coordinate system and reduced to two simple evolutionary equations for the elevation and the velocity potential of the surface. For stationary equations, the proposed approach coincides with the conventional complex variable method. For this case, numerical algorithms for solution of gravity (Stokes) and gravity-capillary wave equations are proposed, and examples of numerical solutions are given. The results imply that gravity-capillary waves do not approach Stokes waves as the capillarity coefficient decreases. Both stationary and nonstationary schemes use Fourier series representation for spatial approximation and the Fourier transform method to calculate nonlinearities. The nonstationary model was validated by simulation of propagating waves with initial conditions obtained as numerical (for gravity and gravity-capillary waves) or analytical (for pure capillary, or Crapper's waves) solutions of the stationary problem. The simulated progressive waves did not change their shape during long-term time integration, which indicates high accuracy of the scheme. Another criterion used for model validation was conservation of integral invariants of simulated multi-mode wave fields. A number of long-term model simulations of gravity, gravity-capillary, and pure capillary waves, with various initial conditions, were performed; for the simulated wave fields, distributions of energy and phase speed over full spectra were analyzed. It was found that the wavenumber-frequency spectra are

<sup>1</sup>OMB contribution no. 131.

l separated into patterns lying along regularly located curves, with most of the energy concentrated along the curves corresponding to free and bound waves. This set of curves can be described by the equation  $D(\omega/n, k/n) = 0$  ( $n = 1, 2, 3, \dots$ ), where  $D(\omega, k) = 0$  is approximated by the linear dispersion relation but does not coincide with it, especially for large  $k$  where there is a tendency for the indicated relation to approximate a straight line. Some other properties of simulated wave fields were also analyzed; these included temporal evolution of the spectra and spatial distribution of the energy of perturbations. The method developed may be applied to a broad range of problems where the assumption of one-dimensionality is acceptable.

### Introduction

Computational techniques for numerical solution of the Navier–Stokes equations have brought new developments to geophysical fluid dynam-

Using modern numerical models, the long-term evolution of several complicated dynamical phenomena in different fluids, including the atmosphere, can be successfully simulated. However, the long-term simulation of a nonlinear multi-mode wave field is difficult to perform, since the best numerical schemes for the Euler equations fail to provide sufficient accuracy for treating nonlinearities in wave motion. The main source of error is primarily due to the finite difference representation of the vertical structure of the flow when waves with different wavenumbers are present. In this case, theoretical and numerical investigations of surface gravity waves are usually based on the equations for potential flow with a free surface. In this case the flow is fully determined by the form of the surface and the velocity potential on the surface and in its vicinity. The potential motion assumption, of course, idealizes the phenomenon, since actual wave motion is both rotational and turbulent. Fortunately, potential theory yields many results which agree well with observations. For example, it is well-known that even linear theory yields phase velocity estimates with an accuracy of the order of 1%. A much more sophisticated theory, dealing with nonlinear wave-wave interactions (Hasselmann, 1962), which is based on the potential motion assumption, gives results which are confirmed by experimental data.

The main advantage of the potential motion approximation is that the system of Euler dynamical equations is reduced to the Laplace equation. However, the solution to the problem of surface wave motion is compli-

ated by boundary conditions which is not the case in the previous system (Cartesian coordinate system). This does not mean that the transformation must be made to calculate the wave field. It is based on the surface of the surface wave applications (Weiss) for a relative convergence of the wave field.

In this case, the wave field was simulated by Higgins and Dold (1962) for steepness, breaking, and extremely large expansion and Sulem's theory of modulation of wave for methods t

Our general 1-D potential scales much is based on